International Journal of Education in Mathematics, Science and Technology

ISSN: 2147-611X

International Journal of Education in Mathematics, Science and Technology (IJEMST)

www.ijemst.com

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Gustavo Morán-Soto[1], Lisa Benson[2]
[1]Instituto Tecnológico de Durango
[2] Clemson University

To cite this article:

Morán-Soto, G. & Benson, L. (2018). Relationship of mathematics self-efficacy and competence with behaviors and attitudes of engineering students with poor mathematics preparation. *International Journal of Education in Mathematics, Science and Technology (IJEMST), 6*(3), 200-220. DOI: 10.18404/ijemst.428165

International Journal of Education in Mathematics, Science and Technology

Volume 6, Number 3, 2018 DOI:10.18404/ijemst.428165

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Article Info

Article History

Received:
07 July 2017

Accepted:
12 January 2018

Keywords

Mathematics self-efficacy
Engineering
Mixed methods

Abstract

This mixed methods research study focuses on two relevant factors in students' decisions to pursue and complete an engineering major: mathematics preparation and mathematics self-efficacy. This study describes the relationship of mathematics self-efficacy on engineering students' performance, behavior, and attitudes in their first college mathematics courses. Participants completed a mathematics self-efficacy survey (n=408); 11 were selected to be interviewed. A grounded theory approach was used to examine participants' behaviors and attitudes in college mathematics courses, their mathematics self-efficacy beliefs, and how these beliefs aligned with their mathematics competence. Interview participants reported relatively high mathematics self-efficacy, but many revealed a mismatch between their mathematics self-efficacy beliefs and mathematics competence levels. Participants who had a balance between these two factors reported being more likely to spend extra time working to overcome their mathematics deficiencies and seeking extra help. However, participants with a mismatch reported being more likely to procrastinate and put little effort into improving their mathematics competence, blaming external factors for their struggles. Despite showing different behaviors and attitudes, all participants reported being likely to continue taking mathematics courses required for their major even after failing their first college mathematics course.

Introduction

Mathematics Self-Efficacy

Self-efficacy beliefs have become central to research into motivational factors that could affect students' interest in pursuing science, technology, engineering, or mathematics (STEM) majors. Findings of these studies show that students who felt that they had good mathematics preparation in their precollege academic experiences, which helped them to develop a high mathematics self-efficacy, were more likely to show interest in pursuing a mathematics-related major like engineering (Hackett, 1985; Lent et al., 1991; Lent, Brown, & Hackett, 1994). The importance of understanding engineering students' mathematics self-efficacy levels can be seen in the different reactions that people normally have when they have low or high self-efficacy for performing specific mathematics tasks. Low mathematics self-efficacy levels have been shown to have an influence on students' decisions to avoid mathematics-related activities that may lead them to have feelings of stress and anxiety (Cooper & Robinson, 1991). Conversely, the literature suggests that after experiencing success in mathematics activities and seeking assistance when they are uncertain, students' mathematics self-efficacy may increase (Bandura, 1986).

This relationship between mathematics self-efficacy and performance has been shown to work both ways, with performance affecting mathematics self-efficacy and with mathematics self-efficacy modifying choices, performance, and persistence in specific mathematics tasks (Williams & Williams, 2010). The influence of mathematics self-efficacy on students' performance in mathematics courses and activities is well-documented, and has been confirmed in different contexts, cultures, and populations (Cooper & Robinson, 1991). The consistency in the relevance of mathematics self-efficacy on not only students' performance but also on their choices and persistence when they are struggling to understand mathematics topics, is a good indicator of the importance of this motivational factor in students' mathematics preparation. Prior research has shown that students' mathematics self-efficacy was lower for students leaving STEM majors, and this factor was more significant for students leaving college during their first semesters (Eris et al., 2010).

The relationship of self-efficacy beliefs with performance could be interactive, with performance affecting self-efficacy and vice versa (Bandura, 1980), and there may be some discrepancies between these two factors during the students' mathematics learning process (Moran & Benson, 2016). These discrepancies may change and evolve according to students' experiences; ideally, students would have a more equilibrated and realistic perception of their mathematics abilities after having enough experience taking challenging mathematics courses (Redmond et al., 2007).

Motivation for this Study

Engineers are important to the economic development and growth of a country (Committee on STEM Edcucation, 2013). Without enough engineers having the necessary skills to design and create new technological solutions to current society needs and problems, a country will face setbacks in its development (National Academy of Engineering, 2015). In spite of the clear importance of training engineers, the projected demands for professionals in STEM fields for the next decade surpasses the projected supply of trained STEM professionals in some countries like the U.S. (Committee on STEM Edcucation, 2013; Snyder & Dillow, 2010). The need for professionals in engineering fields is constantly growing in the biggest economies of the European Union, which currently employ 640,000 to 1.2 million engineering workers in countries like the United Kingdom and Germany, respectively (Erdmann & Schumann, 2010). If countries such as these decrease their current engineering students' graduation rates, they could face a shortage of qualified professionals to fill new emerging engineering jobs.

Understanding how to keep students in their engineering degree programs is of importance to engineering educators around the globe (Chen & Soldner, 2013). The lack of adequate mathematics background for some students who want to become engineers is a great challenge (Kokkelenberg & Sinha, 2010). Once students decide to pursue an engineering major, mathematics competence plays a key role in their academic success. However, there is much variability in the level of high school mathematics preparation for students entering college (Casey, Nuttall, & Pezaris, 1997; Porter, 2011). Once enrolled, mathematics courses like calculus I and II have been identified as barrier courses for engineering students, and struggling to complete these courses could discourage students trying to get an engineering degree (Seymour & Hewitt, 1997; Suresh, 2006). The quality of the mathematics education that some students receive before college could affect their mathematics competence development, and some populations such as under-served groups, minorities, and first generation students are more likely to have some deficiencies in their mathematics preparation due to the lack of opportunities and resources available to them (Flores, 2007; Lee, 2012; May & Chubin, 2003).

Purpose

The purpose of this research is to deepen our understanding of mathematics self-efficacy beliefs for engineering students with poor mathematics preparation, and how these beliefs relate to academic performance, behaviors, and attitudes in their first mathematics course. This research is designed to help inform efforts to decrease engineering attrition, especially for students who experience difficulties with mathematics courses due to a poor mathematical high school background that is reflected in low mathematical competence. Here we are using academic performance as an indicator of competence. Although there is much that is known in general about the relationship between students' mathematics self-efficacy and their performance in college mathematics courses (Hackett & Betz, 1989; Siegel, Galassi, & Ware, 1985; Williams & Williams, 2010), there is little research analyzing the specific case where engineering students start their college mathematics courses with an overestimation of their mathematics self-efficacy. According to mathematics self-efficacy literature, having high mathematics self-efficacy can help engineering students build confidence in their ability to successfully complete a mathematics-related major such as engineering (Hackett, 1985; Lent et al., 1991). However, there is a lack of research analyzing engineering students' reactions to poor performance and struggles understanding the material in mathematics courses due to a poor mathematics preparation despite reporting high mathematics self-efficacy levels. The mismatch between mathematics self-efficacy and competence could negatively affect students choice of behavior and attitudes in mathematics courses (Vancouver & Kendall, 2006), creating a sense of overconfidence in students with higher mathematics self-efficacy beliefs than competence, or generating feelings of anxiety and the need to avoid mathematics related activities in students with lower mathematics self-efficacy levels (Bandura, 1977, 1980).

Engineering students' first experience taking a mathematics course in college could influence their perceptions of their mathematics abilities, and their performance and the mathematics course level have shown to be significant predictors of student retention in engineering majors (Gardner, Pyke, Belcheir, & Schrader, 2007; Middleton et al., 2015; Van Dyken, Benson, & Gerard, 2015). If mathematics educators and professors are more familiar with engineering students' possible reactions to mismatches between their mathematics self-efficacy and competence, then these professors could better prepare to advise students to avoid negative behaviors that may lead them to failure experiences in their first college mathematics courses.

This research was guided by the following research questions:

R.Q. 1 What relationship exists between mathematics self-efficacy of first-year engineering students with poor mathematics preparation and their performance, behavior, and attitudes in their first college mathematics course?
R.Q. 2 What are student behaviors and attitudes in mathematics courses when there is a mismatch between mathematics self-efficacy and competence (low mathematics self-efficacy and high competence; high mathematics self-efficacy and low competence)?
R.Q. 3 What are student behaviors and attitudes in mathematics courses when their mathematics self-efficacy and competence knowledge levels are well matched?
R.Q. 4 What are the effects of a mismatch or a match between mathematics self-efficacy and competence on students' confidence in completing the mathematics courses required for an engineering degree?

For the purpose of this study, student behaviors are defined as the decisions that students make regarding time expended trying to understand their class material, time expended working on their homework assignments, their reaction if they struggle with specific topics, and their decisions to take advantage of or ignore the extra resources available for the class (software, teaching assistants, supplemental instructors, office hours, tutors, etc.). Student attitudes refer to their feelings about mathematics in general and about their current mathematics course.

Despite the importance of mathematics self-efficacy and its influence on engineering students' choices of behavior and performance, the interactions between poor mathematics preparation and mathematics self-efficacy for engineering students have not been studied before. This study will expand current literature by analyzing the different behaviors and attitudes of engineering students with poor mathematics preparation based on their level of mathematics self-efficacy and how well these mathematics self-efficacy beliefs match their actual competence.

Differences in students' mathematics self-efficacy could explain why students with similar cognitive abilities perform at different levels; this difference in performance could be due to differences in the effort that students exerted when they struggled or faced difficulties performing mathematics (Lent et al., 1994). Finding a way to keep engineering students motivated to persist and continue taking mathematics courses despite facing struggles could help engineering educators to address the high attrition rates in engineering majors (Geisinger & Raman, 2013), especially for under-served groups and minorities that are more likely to quit if they face obstacles due to their poor mathematics preparation (Flores, 2007; Lent, Brown, & Hackett, 2000).

Theoretical Frameworks

The concept of self-efficacy emerged in the late 1970's when Bandura (1977) suggested that beliefs in one's abilities to successfully perform a specific task could be a major determinant in one's decisions to attempt that task, and could also affect the effort that one is willing to expend to complete that task when faced with struggles. According to Bandura (1986), self-efficacy refers to "people's judgments of their capabilities to organize and execute courses of action required to attain designated types of performances" (p. 391). Findings of early research in self-efficacy positioned it as an important motivational factor for achievement that may influence students' interest in pursuing specific majors and persisting in their desire to complete that major despite struggling and facing obstacles in the process (Bandura, 1986; Hackett & Betz, 1989; Lent, Lopez, & Bieschke, 1991; Multon, Brown, & Lent, 1991).

Developed by Bandura (1986), Social Cognitive Theory highlights cognitive, vicarious, self-regulatory, and self-reflective processes that modify people's behavior and choices. Within Bandura´s social cognitive constructs, self-efficacy has a key role predicting one´s choice of activities, effort expenditure, persistence, and emotional reactions when facing difficulties performing any particular task. These beliefs are task-specific and one's skills in a particular task may influence only the individual performance on that task (Bandura, 1986). It is important to understand that self-efficacy beliefs could be related to objectively assessed skills, but these two factors could also be very different depending on one's experiences and circumstances.

Bandura's theory hypothesizes that self-efficacy beliefs that slightly exceed one's current skill level could encourage people to try challenging activities that may promote better skill development. On the other hand, self-efficacy beliefs that grossly exceed or underestimate current skill level could lead to maladaptive behavior; a discrepancy with high self-efficacy without good skills that support these beliefs could lead to failure or poor performance in challenging activities, while pessimistic beliefs or lack of confidence in well-developed skills may prompt avoidance of activities that are within one's competence range.

Research Design

A sequential explanatory mixed methods design was selected to conduct this research (see Figure 1) (Creswell & Clark, 2011). Although the quantitative phase was performed first, there was an emphasis on the qualitative phase of this mixed methods design. To get a thorough description of students' experiences taking college mathematics courses, a constructive grounded theory methodology was followed in the qualitative phase of this research to develop a theory based on the data (Charmaz, 2006). The sequential explanatory design allowed the researchers to elaborate and expand the conclusions about the participants' mathematics self-efficacy levels reported during the quantitative results. By comparing the quantitative results with the qualitative findings, the researchers were able to find and further explain possible relationships between students' mathematics self-efficacy levels and their chose of behavior and attitudes in mathematics courses.

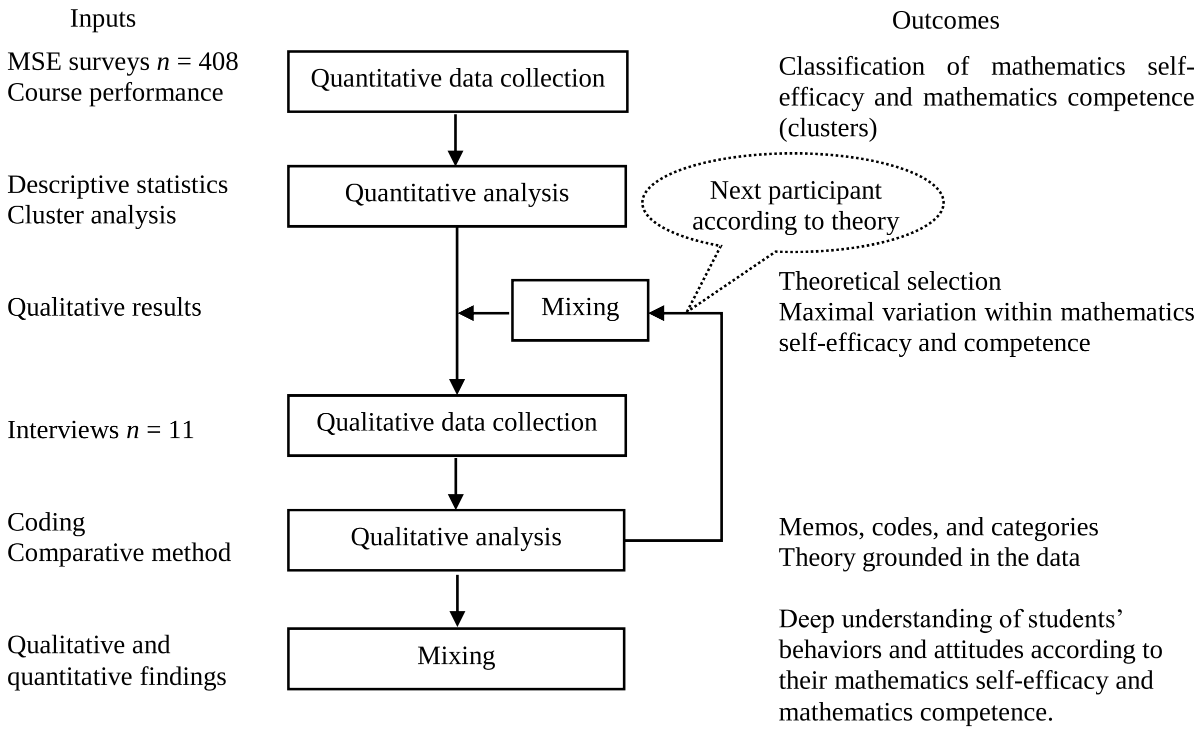

Figure 1. Inputs and outcomes of the sequential explanatory mixed methods design

A grounded theory approach was selected for the qualitative phase of this study due to the lack of literature and prior studies about mathematics self-efficacy of engineering students with poor mathematics preparation (Corbin & Strauss, 2008). Developing a theory based on students' experiences is an important step for engineering and mathematics educators seeking to decrease engineering attrition caused by poor mathematics preparation. Designing mathematics courses and possible interventions to help engineering students understand how their mathematics self-efficacy beliefs might influence their behavior and performance in mathematics courses if these beliefs are not aligned with their mathematics abilities could help students identify and avoid struggles and failing experiences in mathematics courses.

Participants

Participants for this study were selected based on the mathematics course in which they were enrolled during their first year at a southeastern United States university. Students who have the lowest scores on the university mathematics placement test are placed in a precalculus class ("Precalc"). Those scoring slightly higher on the placement test but who are not ready to take a full semester of calculus as their first college mathematics course are placed in a two-semester course combining precalculus and calculus. The first semester course in this series is referred to as "Long Calc," and the following course is "Long Calc II." Both Precalc and Long Calc are considered non-college-level mathematics courses and are designed to help prepare students with poor mathematics backgrounds to succeed in calculus courses.

Students from all majors enrolled in Precalc and Long Calc for the fall and spring semesters over the course of one academic year were stratified (Onwuegbuzie & Collins, 2007), aiming to collect information about students with poor mathematics preparation in their first college mathematics course. A total of 408 students participated in the quantitative phase of this study: 101 students in three sections of Precalc and 207 students in 5 sections of Long Calc in the fall semester; and 20 students in one section of Precalc and 80 students in 3 sections of Long Calc in the following spring semester. Six participants were randomly selected for interviews from the Precalc and Long Calc courses, and these interviews were analyzed, before a theoretical sampling could be conducted, in keeping with a grounded theory approach (Bryant & Charmaz, 2007). Preliminary findings from these six participants formed the baseline of the emerging theory that helped refine the research questions and modify the study's research design. This method of participant selection is in line with the first steps of a grounded theory approach, where there is not enough data to perform theoretical selection (Bryant & Charmaz, 2007). Preliminary findings of this first stage of the grounded theory approach indicated the need for a more accurate measurement of participants' mathematics self-efficacy beliefs for comparison with their mathematics competence and to enable theoretical selection based on the mathematics self-efficacy level.

Following a sequential data collection design (Onwuegbuzie & Collins, 2007), five additional participants were purposely selected from the previously stratified sample following the guidelines determined by the theoretical sampling after the analysis of quantitative survey data from prior participants. These five participants' data was added to the previous six participants' (n = 11) data to be analyzed following a grounded theory approach. The last five participants of the qualitative phase of this study were selected with the goal of reaching maximum variation within students' mathematics self-efficacy beliefs and competence levels to develop a thorough theory (Teddlie & Yu, 2007).

Quantitative Phase Methods

Data Collection

This research used the Mathematic Self-Efficacy Survey (MSES), developed by Betz and Hackett (1983), to measure participants' mathematics self-efficacy for the theoretical selection. The MSES consists of 52 items measuring three different mathematics self-efficacy subscales: mathematics problem-solving (18 items), everyday mathematics tasks (18 items), and mathematics courses (16 items). Participants rated their level of confidence performing different mathematics-related activities for each question on a scale ranging from no confidence at all (0) to complete confidence (10). The survey was selected for this study due to its high reliability in measuring mathematics self-efficacy in prior studies (e.g. Brown & Burnham, 2012). Both the full-scale original survey and its subscales independently have been examined previously with coefficient alpha values ranging from 0.90 to 0.96 (Brown & Burnham, 2012; Kranzler & Pajares, 1997; Lent, Lopez, & Bieschke, 1991; Pajares & Miller, 1995). These Cronbach's alpha values demonstrated the internal consistency reliability of the MSES for different populations and contexts, providing confidence that the survey could accurately measure engineering students' mathematics self-efficacy for this research population. Some MSES items were adapted to the context and population of this study aiming to get a self-efficacy measurement that could be related to participants' performance in these specific courses (Bandura, 1986). For example, an item asking students about their level of confidence working with a slide rule was modified to include a scientific calculator (CASIO fx-350MS) instead of the outdated slide rule. These changes helped participants to interpret the survey questions and relate these questions to the mathematics activities that they perform in their daily lives.

Students from all majors at the university in the sections of mathematics courses in which instructors agreed to distribute the MSES completed the survey during class time. Instructors distributed and collected the paper-based surveys at the end of their class, and they returned them immediately to the researcher for data analysis. The six participants who were interviewed before the mathematics self-efficacy survey was used for theoretical sampling purposes did not complete the survey; these six participants were missing this quantitative mathematics self-efficacy measurement.

Participants' mathematics self-efficacy level was determined using both survey and interview responses. An average of participants' interview responses about their mathematics self-efficacy for different mathematics related activities from 1 to 10 (e.g., How confident do you feel that you can solve your homework after being taught a difficult topic? Could you select your level of confidence in a scale between 1 (not at all) and 10 (very confident)) were used as their mathematics self-efficacy level for the mixed analysis to be compared with participants' mathematics competence. All participants provided a mathematics self-efficacy estimate as part of their interview; the number of interview questions that participants answered to get that mathematics self-efficacy estimate changed from interview to interview because of the evolving nature of the grounded theory methodology. The seventh and all subsequent interview participants completed the MSES to classify their mathematics self-efficacy beliefs before the interview. Results of the MSES were used to purposely select the most appropriate participants to complement and further develop the emerging theory. Only the last five participants answered the MSES, and these survey averages were used for the purpose of theoretical selection only.

Participants' mathematics competence was estimated based on their performance in their college mathematics course at the time of the interview. Participants' performance information was obtained from the online course software used to teach the course for the Precalc students, and from the course test scores for the Long Calc students. Some grades were not final due to the time of the semester in which the interview was conducted, so interim grades were used. The competence scores were on a scale from 1 to 100, so the mathematics competence estimation was calculated by dividing the interim grades by 10 to be compared with participants' mathematics self-efficacy beliefs for the mixed analysis of the data.

Data Analysis

The data collected with the MSES (n=408) were analyzed using descriptive statistics and cluster analysis to classify participants' mathematics self-efficacy level using the statistical software R (Team, 2012). A k-means cluster analysis (Maechler, Rousseeuw, Struyf, Hubert, & Hornik, 2015) was conducted to classify participants in three homogenous subgroups based on their mathematics self-efficacy levels for each semester that the MSES was used to collect data. K-means is a partitioning type of clustering method and is appropriate for cases in which the number of clusters can be assumed based on knowledge of underlying theory and prior results (Ehlert, Faber, Benson, & Kennedy, 2017). The three clusters identified through the k-means cluster analysis classified participants' mathematics self-efficacy beliefs as medium-low, medium-high, and high according to their average MSES scores. These clusters were used to purposefully select individuals with the appropriate characteristics for the qualitative phase interviews, aiming to get a maximum variation within mathematics self-efficacy beliefs. Participants' mathematics competence was classified as high or low according to course performance data.

An additional mathematics self-efficacy average was calculated using the data from the participants' interview responses in which they rated their confidence in performing mathematics related activities. This mathematics self-efficacy average was used for the mixed analysis of the reasons behind participants' behaviors and performance within different mathematics activities. All eleven interviewed participants reported a mathematics self-efficacy average from their interviews, and the last five participants were selected based on the MSES results and emerging theory. The difference between these two mathematics self-efficacy measurements is that interview data was used as the participants' mathematics self-efficacy average during the mixing phase, and the MSES data were used for theoretical selection purposes only.

Quality Considerations

The adapted version of the MSES was validated using Cronbach's alpha values to evaluate the internal consistency reliability of the survey items for this specific population (Thorndike & Thorndike-Christ, 2010). All MSES constructs were expected to have a Cronbach's alpha value above 0.7 (Nunnally & Bernstein, 1994), showing that they are measuring similar theoretical concepts. Content and face validity were also evaluated by asking qualitative experts and mathematics instructors at the university about the clarity and possible students' interpretation of the survey items. This feedback guided modifications and rewording of some survey items to ensure the items were understandable for participants and were all measuring the same construct in a thorough way (Creswell, 2009).

Qualitative Phase Methods

Data Collection

The data for the qualitative phase of this research were collected through semi-structured interviews, including questions about participants' reasons for choosing engineering, persistence in their desire to become engineers, mathematics self-efficacy based on Bandura's guide for constructing self-efficacy scales (Bandura, 1986), and social supports (Lent et al., 1994). During the interview, participants were asked to rate their self-efficacy on a scale of 1-10 for specific mathematics skills, such as solving word problems and finding errors in their calculations. The interview questions related to mathematics self-efficacy were designed to let the participants express their feelings and rate their level of confidence performing mathematics-related activities. The questions used the phrase "level of confidence" instead of mathematics self-efficacy to avoid misunderstandings with respect to the term "self-efficacy." Self-efficacy interview questions focused on challenging mathematics activities and common assignments that students normally face in their college mathematics course, with questions such as: "How confident do you feel that you can do the most challenging mathematics exercises of your class if you have all the time and resources you need to work to your satisfaction? How is your level of confidence in your mathematics abilities when you are solving mathematics problems in class compared to solving mathematics problems on a test?" At the end of every mathematics self-efficacy question, the participants were asked to rate their mathematics self-efficacy using a scale suggested by Bandura (1986) ranging from 1 to 10, with the following question: "What is your level of confidence for this question on a scale from 1(not at all) and 10 (very confident)?"

The interview protocol was modified as needed at certain points during the qualitative data collection to expand participants' responses relative to previous participants. The last five interview participants were purposefully selected according to the quantitative phase results, with the goal of maximum variation in mathematics self-efficacy. Each participant was selected after analyzing and coding the data from the previous participant; theoretical selection was performed based on the emerging codes and categories from previous participants. Eleven participants were interviewed between the Fall 2015 and Spring 2016 semesters.

Data Analysis

Following a constructivist grounded theory approach, the qualitative data was initially coded line-by-line using open coding (Charmaz, 2006). Codes emerged directly from the data, and they were named using gerunds and participants' words (in vivo coding) during the interview, in order to develop a theory that could accurately represent participants' experiences and actions in their college mathematics courses (Bryant & Charmaz, 2007). The first interview was coded and analyzed before conducting the next interview (Bryant & Charmaz, 2007), and this process continued for all subsequent interviews. The codes, memos, and responses from the interviews analysis were used to modify the interview protocol according to emerging themes and missing information before conducting the next interview.

This coding philosophy facilitated the theoretical interpretation of the data based on what was reported by the participants. The initial codes were constantly analyzed and compared between participants. During this analysis process, memos were written as notes that described the researcher's interpretation of the emerging codes (Charmaz, 2007). These memos helped the researcher develop categories based on the constant comparison of codes, memos, and data, allowing the categories to emerge as the more representative actions of engineering students with poor mathematics preparation in college mathematics courses. After a constant comparison of codes, memos, categories, and data from different participants, the final categories became the core of a theory grounded in the data that described mathematics self-efficacy perceptions of engineering students with poor mathematics preparation, and how their self-efficacy beliefs are related to their behavior and attitudes in college mathematics classes. Preliminary findings of this qualitative analysis were used together with the quantitative results to inform the theoretical sampling. This theoretical sampling and continual refining of the interview protocol were part of a cycle for selecting the next, most appropriate participant.

Quality Considerations

According to Johnson (1997), there are five types of validity that need to be considered to validate qualitative studies: descriptive validity, interpretive validity, theoretical validity, internal validity, and external validity. Three of these five types of validity were applied in this study. Following Johnson's validation framework an additional person was present in the interviews to take notes and help triangulate and discuss the data after the interviews, which provided descriptive validity. Additionally, participants' feedback was used to demonstrate interpretive validity and accuracy of the conclusions. Theoretical sampling to purposefully select specific types of participants, which enriched and complemented the emerging theory, provided theoretical validity.

Mixed Phase Methods

Data Analysis

The mixing between the quantitative and the qualitative phases occurred in two different places in this research. The first mixing phase took place during data collection. During this mixing phase the quantitative results informed the participant selection for the qualitative phase to perform theoretical sampling accurately, using the MSES results to purposefully select the most adequate characteristics of the subsequent participants for the qualitative phase (Sandelowski, Voils, & Knafl, 2009). Additionally, after finalizing both quantitative and qualitative phases, the findings and data of these two phases were analyzed together, aiming to expand our understanding of the possible influences of mathematics self-efficacy on engineering students and answer the research questions. This final analysis thoroughly integrated the two methods, helping the researcher consolidate, compare, and integrate into a coherent whole both qualitative and quantitative findings (Onwuegbuzie & Combs, 2010). To accomplish this, survey data (clusters) and interview data (categories, codes, diagrams) were compared side by side for each participant, aiming to triangulate the data to compare and expand the findings of each phase (Moran-Ellis, 2006). This second mixing phase helped the researcher to classify participants in different groups depending on their mathematics self-efficacy beliefs and mathematics competence relationship. If participants showed a gap larger than 1 point between their mathematics self-efficacy beliefs assessment (between 1 and 10) and their mathematics competence assessment (between 1 and 100 divided by 10), then these participants were considered having a mismatch between these two factors; if the gap between these two factors was lower than 1 point, then these participants were considered having a match between their mathematics self-efficacy and competence (see Table 2).

Quality Considerations

This study used the Onwuebuzie and Johnson legitimation framework (2006) for mixed methods quality considerations. For this framework, nine legitimation types are considered to improve the quality of mixed methods studies, including: (1) sample integration, (2) inside-outside, (3) weakness minimization, (4) sequential, (5) conversion, (6) paradigmatic mixing, (7) commensurability, (8) multiple validities, and (9) political. Five of these nine legitimation types were applied in this study. The sample for the qualitative phase was selected from a subset of the quantitative sample by using the quantitative results to inform the theoretical sampling for the qualitative phase, which provided sample integration and sequential legitimization. The relationship between participants' behaviors and attitudes and their mathematics self-efficacy level were described using the richness of the data collected in the qualitative phase. These findings were complemented with quantitative results that helped to develop a thorough theory and minimized demonstrated weaknesses. Data conversion (quantitizing nor qualitizing (Sandelowski et al., 2009)) was not used to analyze the whole data, but a constant side-by-side data comparison was conducted before making inferences. Finally, individual quantitative and qualitative quality assessments were performed prior to the final integration of the final results to provide multiple sources of validity.

Results

Quantitative Results

Almost all the MSES's constructs showed good internal consistency reliability in the fall survey, with Cronbach's α values above 0.8 (see Table 1). The only construct that was slightly below the acceptable cutoff of 0.7 (Trochim & Donnelly, 2007) was the mathematics activities construct for the spring semester, with $\alpha = 0.69$.

Although the mathematics activities construct had a low Cronbach's α value for the spring semester, this construct showed good internal consistency reliability in the fall with α = 0.91. This mathematics activities construct was included in the final data analysis due to the smaller sample in the spring semester (n = 100), which is less reliable than the fall Cronbach's α value for the same construct using a bigger sample (n = 308) (Onwuegbuzie & Leech, 2007).

Table 1. Cronbach's α values of the items of each construct on the MSES

Semester	MSES Construct	Cronbach's α
Fall 2015	Mathematics Activities (18 items)	0.91
	Mathematics Courses (14 items)	0.83
	Mathematics Problems (18 items)	0.90
Spring 2016	Mathematics Activities (18 items)	0.69
	Mathematics Courses (14 items)	0.80
	Mathematics Problems (18 items)	0.90

Participants' Mathematics Competence and Mathematics Self-Efficacy

Table 2 summarizes participants' mathematics competence knowledge based on their performance in the mathematics course that they were taking at the time of the interview and their mathematics self-efficacy levels for both the survey and the interview.

Table 2. Comparison between participants' mathematics self-efficacy and their mathematics competence

Participants	Mathematics self-efficacy		Mathematics	Comparison	
	Interview	MSES	Competence	Self-efficacy	Competence
P Spring 2014	6.6	NA	8.1	Medium	Higher
L Summer 2014	7	NA	9.1	Medium	Higher
P Fall 2014	7.6	NA	1.5	High	Lower
P Fall 2014	8	NA	2.9	High	Lower
L Spring 2015	6.2	NA	7.3	Medium	Higher
P Spring 2015	6.8	NA	6.7	Medium	Matched
L Fall 2015	7	5.8	8.5	Medium	Higher
L Spring 2016	6.5	7.9	6.6	Medium	Matched
P Spring 2016	9	9	8.3	High	Matched
L Spring 2016	6.9	8.8	6.5	Medium	Matched
L Spring 2016	7.8	8.6	8.2	High	Matched

P = Precalc; L = Long Calc; NA is for the six participants that did not take the MSES.

The k-means cluster analysis separated all participants enrolled in an engineering major in three different clusters based on their mathematics self-efficacy level. The average MSES score (ranging from 1 – 10) was used to cluster participants into three groups for the semesters that the MSES was distributed and analyzed. These groups were labeled medium-low (*n* = 24 for the fall and *n* = 6 for the spring), medium-high (*n* = 37 for the fall and *n* = 13 for the spring), and high (*n* = 31 for the fall and *n* = 5 for the spring) mathematics self-efficacy groups (see Table 3). None of the three groups was labeled as low due to the relatively high confidence that participants reported about their mathematics abilities on the survey. The lowest mathematics self-efficacy mean found by the k-means cluster analysis was 6.3, and this value was not low enough to be considered as poor mathematics self-efficacy by the researchers. The low mathematics self-efficacy label was purposely avoided to emphasize the high mathematics self-efficacy levels in general and facilitate interpretation of the final findings and conclusions. The overall high mathematics self-efficacy levels and the lack of participants reporting a low mathematics self-efficacy were considered enough evidence to classify these participants as having relatively high mathematics self-efficacy levels. This relatively high mathematics self-efficacy included participants with medium and high mathematics self-efficacy levels, emphasizing the confidence that most students reported about their mathematics abilities.

Table 3. Mathematics self-efficacy survey mean scores separated in 3 clusters by the k-means cluster analysis

Semester	Clusters	Mean (MSES)	Students	Participants Selected
Fall 2015	1 → Medium Low	6.6	24	1
	2 → Medium High	8	37	0
	3 → High	9.2	31	0
Spring 2016	1 → Medium Low	6.3	6	0
	2 → Medium High	8	13	2
	3 → High	9.3	5	2

Following theoretical selection guidelines (Corbin & Strauss, 2008), participants were selected from each of the three mathematics self-efficacy groups to complement and test the emerging theory. One participant was selected from the medium low mathematics self-efficacy group for the fall semester, and two participants were selected from both the medium high and high self-efficacy groups for the spring semester (see Table 3). Theoretical sampling was performed to reach maximum variation within these three groups (Teddlie & Yu, 2007). However, it was difficult to reach theoretical saturation due to the lack of participants reporting low mathematics self-efficacy levels during the survey and some difficulties recruiting participants from the required groups to keep developing the theory. These difficulties and theoretical saturation issues are described as limitations for this study later in this paper.

Qualitative Findings

Summaries of each of the 11 participants' interviews were developed after analyzing all of the qualitative data. These summaries describe participants' behavior during their experiences with college mathematics courses, classifying their experiences in terms of how they reacted to different situations, and contrasting these for different mathematics self-efficacy and competence levels. After a constant comparison between participant summaries and a deep analysis of all qualitative data, codes, and memos, the following categories were established to describe participants' behaviors and attitudes: persisting in college mathematics courses, adjusting mathematics self-efficacy, failing a college mathematics course, refreshing basic mathematics competence, spending extra time working on mathematics, taking struggles as opportunities, seeking help, lacking college-level mathematics experience, feeling overconfident, lacking effort, blaming other factors, and experiencing stress working on mathematics.

These categories were the baseline of the theoretical ideas (Charmaz, 2008b) used to develop a theory that described students' behavior and attitudes in their college mathematics courses, and identified differences between students with matched mathematics self-efficacy and competence and students showing a gap between these two factors based on the results of the mixed methods analysis.

Mixed Methods Results

All participants reported mathematics self-efficacy ranging from medium to high, and they revealed high confidence about their general mathematics abilities during both their interview and their qualitative measurements (see Table 2). High mathematics self-efficacy beliefs could influence students' decision to pursue an engineering major (Lent et al., 1991; Siegel et al., 1985); thus it was expected that these participants indicated high levels of confidence in their mathematics skills. Despite participants' strong confidence in their mathematics abilities, some of them revealed having some deficiencies in their mathematics competence after the mixed analysis comparing their interview responses with their mathematics grades (see Table 2). Other participants revealed a better mathematics competence that they thought they had. Thus, they were rating their mathematics self-efficacy lower than the actual mathematics competence they possessed (see Table 2). This mismatch between participants' mathematics self-efficacy beliefs and their real competence affected these participants' behavior and attitude in mathematics courses, making them more likely to behave in certain ways depending on the direction of the mismatch.

Some participants grouped together after analyzing and comparing their mathematics self-efficacy beliefs with their mathematics competence (see Figure 2). Participants that clustered together showed similar behaviors and attitudes while taking college mathematics courses, and these behaviors and attitudes were used to refine and expand the categories described in the qualitative results. These refined categories were the key to developing a theory that explains how mathematics self-efficacy of first-year engineering students with poor mathematics preparation influences their performance and behavior in college mathematics courses.

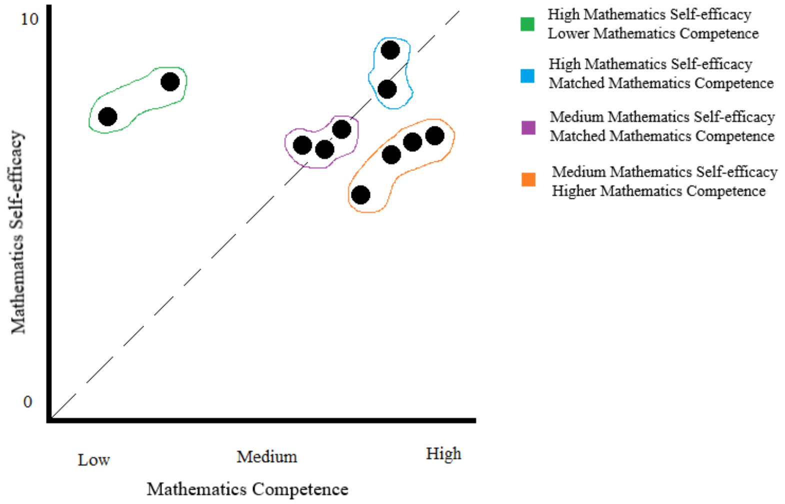

Figure 2. Participants' mathematics self-efficacy compared with their mathematics competence and how they grouped according these two factors

Participants' behaviors based on their mathematics self-efficacy and mathematics competence relationship

Specific behaviors and attitudes described by participants are categorized for the four groups of participants with similar mathematics self-efficacy and competence levels, as shown in Figure 2.

Mismatch between High Mathematics self-efficacy and Lower Mathematics Competence

Participants in this group demonstrated overconfidence about their abilities to perform well in their mathematics courses. They normally struggle to complete their mathematics assignments on time because they believe they will not need much time to understand their mathematics course material than they really need:

> I wasn't putting in all the time [on my math course], and I wasn't practicing enough. So like I said when I went into the test, even if I could do the problems, I went into all four tests being able to do the problems, but not being able to do them fast enough to finish the test on time.

It is also typical for this type of engineering student to struggle during their first experience taking a college mathematics course, and they normally blame external factors for their poor understanding of the class material, such as the course format or the lack of an experienced professor to set the class pace and explain challenging topics:

> It's online [the math course] so that's my biggest problem. It's all taught online, so that's not really my style. So I'm a little bit behind in like the pace they want me to be at, but as for understanding the material I'm doing pretty well.

These participants started their mathematics college classes believing the mathematics material would be easy to understand because they had developed the belief that mathematics courses are not very challenging based on their high school experiences. Students who avoid taking calculus in high school may have an inflated sense of their mathematics abilities because of their experience taking basic mathematics topics that did not challenge their mathematics understanding. Although current literature suggests that a high mathematics self-efficacy may have a positive impact on students' behaviors in mathematics classes (Gore, 2006; Schunk, 1991), engineering students with high mathematics self-efficacy need to be cautious about thinking that their mathematics abilities are better than their mathematics competence, especially if their beliefs are based on their performance in basic mathematics courses. If these students' mathematics competence is considerably lower than their mathematics self-efficacy beliefs, they could experience setbacks in their mathematics education once they try to perform well in college-level mathematics courses such as calculus with the wrong idea that they do not need to spend extra time working to improve their mathematics abilities.

Engineering students with a mismatch between high mathematics self-efficacy and lower mathematics competence are likely to experience feelings of stress and anxiety when struggling on their mathematics material because they believe they should be able to learn the course material and perform well without much effort. One participant noted,

> I'm relatively OK until I get a problem wrong, and then, when I rework it and it's wrong in a different way, then I get like really stressed because I tried it in two different ways and it didn't work.

The stress and feelings of anxiety usually led these participants to stop working on their mathematics material, which in turn made them fall behind and prioritize other academic activities instead of trying to improve their mathematics abilities (Jameson, 2013; Meece, Wigfield, & Eccles, 1990). Although engineering students of this type are likely to fall behind schedule in their mathematics course and experience failure for the first time due to their overconfidence about their mathematics abilities, our participants in this group took this failing experience as an opportunity to refresh their basic mathematics knowledge. This positive attitude about failing their first college mathematics course helped these participants maintain high confidence in their mathematics abilities so they could complete the same course the following semester:

> Since I'm taking the class again, I'm putting a lot more effort into it, a lot more practice, um, and like really being involved with the course.

Realizing that their high school mathematics preparation was not as strong as they believed may influence their mathematics self-efficacy beliefs in future college mathematics courses:

> I knew I've always been good at mathematics from like elementary school up to high school, but now it is getting in more depth and in the hardest stuff I kind of... kind of don't feel like I'm that good anymore.

Students will more likely approach the course with a more realistic view of their mathematics abilities and a better understanding of the time that they would need to spend on their mathematics material to complete the course.

Mismatch between Medium Mathematics self-efficacy and Higher Mathematics Competence

Participants with medium mathematics self-efficacy and higher mathematics competence were more reluctant to get involved in challenging mathematics activities or to take advanced college mathematics courses for which they did not feel prepared (Manley & Rosemier, 1972), even when some of them have taken courses like calculus in high school. These students' decision to start in a non-college-level mathematics course, even though they may have been prepared for higher levels, may be because they are trying to refresh their prior mathematics knowledge before taking a college-level mathematics course:

> I'm now taking Precalc, just because I want. Also because I just want to make sure of my basic skills before I try advance math courses. You don't want to try and push your way through this math, and maybe pass it, but not really get it.

They are usually confident enough about their mathematics abilities to sufficiently complete the mathematics courses required by their engineering major, but they consciously decided to be cautious about the possibility of struggling with more advanced mathematics topics in advanced mathematics courses. These participants viewed struggles in learning mathematics as opportunities to test their real understanding and improve their mathematics abilities, considering struggling as a normal part of learning advanced mathematics topics. Although these participants believe that college mathematics courses were more challenging than their high school courses, they were likely to have positive behaviors and attitudes in their college mathematics classes, spend extra time working on their mathematics assignments, seek help in case they struggle to understand any mathematics topic, and remain motivated to improve their mathematics abilities even when they were facing struggles in understanding new mathematics topics:

> Usually I would go back through the homework assignments that we have done. And also there are extra assignments that we don't get to in class, so I'll just go back and practice those, and if I have a problem with one, then I'll go to my teacher and say can you help me with this and she will guide me through it.

These engineering students normally put in more effort to learn new and challenging mathematics material, but they are likely to wait until they get help from someone to continue working on a mathematics problem they fail to solve initially on their own:

> When I get helped, it just helps me to remember more, 'cause when you go to ask someone you kind of recall the conversation. I do office hours definitely with the teacher, but if it is a minor homework problem. I go with my friends, or my roommate, or anyone that I know they are good at math, or they are in a higher level of math than I am. So you know they will know the problem or the topic I'm working on and they can help me.

Although seeking help before giving up helps these students believe that they can complete advanced mathematics courses if they find the right support systems, these beliefs may also generate poor confidence in their own mathematics abilities when they are working on their own and there is no one to help them, such as on an exam, in case they struggle:

> In class I'll say about an 8 [mathematics self-efficacy] and in a test I'll say about a 5 [mathematics self-efficacy]. In class I just ask for some help and then I understand it, and during the test, you know, they [mathematics problems] don't look the same to me, or I don't see them as the same and I just get confused. I kind of like… I just answer what I really know how to answer.

This type of student is more likely to look for a mathematics course that could let them adapt to the different teaching style and difficulty level of college mathematics courses without feeling rushed or getting behind when transitioning into college.

Matched High Mathematics self-efficacy and High Mathematics Competence

Participants with matched high mathematics self-efficacy and mathematics competence demonstrated positive behaviors and attitudes in college mathematics courses. Such behaviors included spending extra time working on their mathematics material, seeking extra help from tutors, and attending office hours (Zimmerman, 2000):

> I´m normally done with my classes by twelve, so I'm at the library doing work or in my room doing work with a friend. I do work at least an hour a day, at least one hour per day for math class if not more.

These engineering students usually view their struggles learning mathematics as part of the mathematics learning process, and they consider these struggles as opportunities to improve their mathematics competence (Lent et al., 1991). They enjoy the challenge in learning new and/or advanced mathematics topics:

> I feel like things [mathematics courses] are getting harder sometimes, but as long as you want to learn, and you want to get better then you can do it. And I've always loved math so I always try to be better at it.

Students of this type are very confident about completing the mathematics courses required by their engineering majors, and they usually show the will to take advanced mathematics courses more than once if they struggle to complete the course the first time (Multon et al., 1991; Suresh, 2006):

> I will [finish all math courses required by my major] even if that takes an extra semester, or a year of college. I think I would because this [engineering] is for sure what I want to do.

All participants in this category were taking their mathematics course for the second time after failing their first college mathematics course. They reported changing their approach to taking a college mathematics course after experiencing setbacks and failure the first time they took the course. They used this failing experience as a wake-up call that helped them to realize they would need to spend extra time working on their mathematics material; they sought help to improve their mathematics abilities to stay on pace with the class at the college level:

> I didn't pass the first part of the Long Calc. It was kind of... kind of a reality check, 'cause I guess I could have studied more, went to tutoring more, and stuff to make my grade better in that class.

These participants did not lose confidence in their mathematics abilities after failing their first college mathematics course, but rather they used their failing experience to match their real mathematics competence with more realistic mathematics self-efficacy beliefs. Keeping a high mathematics self-efficacy that was better aligned with their mathematics competence helped these participants feel motivated to work harder the second time that they took the same mathematics course (Bandura & Schunk, 1981; Walker, Greene, & Mansell, 2006). This result was confirmed by their improved performance when taking the same mathematics course that they failed the semester before.

Matched Medium High Mathematics self-efficacy and Medium High Mathematics Competence

Participants who had a matched medium high mathematics self-efficacy and mathematics competence lacked college-level mathematics skills coming from high school, and that poor understanding of advanced mathematics topics was a problem when they took their first college mathematics course. These engineering students usually think their mathematics competence is acceptable to pass the course without working hard to improve their mathematics abilities causing them to remain in a comfort zone where they overlook the importance of hard work and practice in mathematics courses (Lent et al., 1984):

> Well the first time [taking a college math course], I was under the assumption that 'Oh I know this stuff, I don't have to study as much as thought I did,' and I didn't put as much practice as you need to do it.

Participants in this group typically blamed external factors such as the professor's teaching style and the fast class pace for their failing experiences in college mathematics courses, but they usually showed limited interest in spending extra time working on their own to improve their mathematics abilities.

All participants in this category failed their first college mathematics course. After this failing experience, these engineering students realized they would need to work harder and spend more time working on their coursework to be able to complete all the mathematics courses required by their engineering majors.

> Failing the course made me feel pretty bad, and after that I told myself I will work better the next time, so I can pass it and not have that feeling again.

Their failing experience showed them that their mathematics courses were more difficult than expected, and they would need to lower their mathematics self-efficacy beliefs to align with their ability. This mathematics self-efficacy adjustment brought their mathematics self-efficacy beliefs closer to their real mathematics competence, making these students more likely to seek help when struggled with mathematics topics (Lopez & Lent, 1992). Although these participants showed more interest on improving their mathematics abilities the second time they took a college mathematics course, they remained more likely to stop trying to solve mathematics problems if they were struggling and there was no one there to help them understand what was wrong with their approach. They were likely to experience stress if they struggled working on mathematics problems:

> I don't even think that I hate math, 'cause I enjoy talking about mathematical concepts with professors. I just hate the struggle. I hate when you can't do it, and you sit there over ten online homework problems for hours and is like... if I could do this in 20 minutes, math would be my favorite subject.

However, they reported to be likely to persist in repeating mathematics courses they had failed, viewing their failing experiences as opportunities to refresh their mathematics competence and perform better next time.

Discussion

The population of engineering students in this study reported relatively high mathematics self-efficacy, especially when they were referring to their abilities to perform mathematics in general and their confidence in completing all the mathematics courses required by their majors. These results support current literature about students who choose engineering as their major feeling confidence that their mathematics competence will be good enough to perform well in college mathematics-related activities (Hackett & Betz, 1989; Lent, Lopez, & Bieschke, 1991.) Most of the participants in this study were very confident that they would successfully complete their first college mathematics course, even when they showed poor mathematics preparation coming from high school. The gap between high mathematics self-efficacy and lower mathematics competence was related to participants' negative behaviors and attitudes in mathematics courses such as feeling overly confident about their mathematics competence, blaming other factors for their struggles and failing experiences, and not putting forth effort to improve their mathematics abilities (see Figure 3). These negative behaviors and attitudes created a stressful environment in the participants' mathematics courses, making them more likely to perform poorly and fail their first college mathematics course.

Having a failing experience in their first college mathematics course put participants in a position where they had to adjust their mathematics self-efficacy to a lower level, closer to their real mathematics competence. Their mathematics self-efficacy remained high enough to make them think that they still could complete all the mathematics courses required by their majors, but after failing their first college mathematics course, they were more likely to adjust their mathematics self-efficacy to match their actual mathematics competence. This new and more realistic perception of their mathematics competence was related to a noticeable change in participants' behavior and attitudes when taking a college mathematics course the following semester. Participants with mathematics self-efficacy beliefs aligned with their mathematics competence were more likely to have positive behaviors and attitudes in mathematics courses such as spending extra time working on their mathematics material and seeking help to address their doubts, and viewing their struggles in learning new mathematics topics as opportunities to develop better their mathematics abilities and gain a deeper understanding of the course material (see Figure 3). By adjusting their approach in the same mathematics course, participants were better able to develop their mathematics competence and they usually enjoyed the challenge involved in learning new difficult mathematics topics.

Although some participants reported struggling with the pace of the course and understanding advanced mathematics topics in their first college mathematics course, their mathematics self-efficacy remained always relatively high. This medium to high mathematics self-efficacy level was related to participants' confidence about satisfactorily completing all the mathematics courses required by their majors if they worked hard and put in extra effort to improve their mathematics abilities. This result is supported in the self-efficacy literature where students feel more motivated to put extra effort to overcome difficulties when they are facing challenging activities if they believe they can succeed performing such activity (Bandura, 1980; Marra, Rodgers, Shen, & Bogue, 2009; Walker et al., 2006). These engineering students were confident about their abilities to complete their college mathematics courses even if they would need to repeat some of the most advanced courses due to failing to pass them the first time.

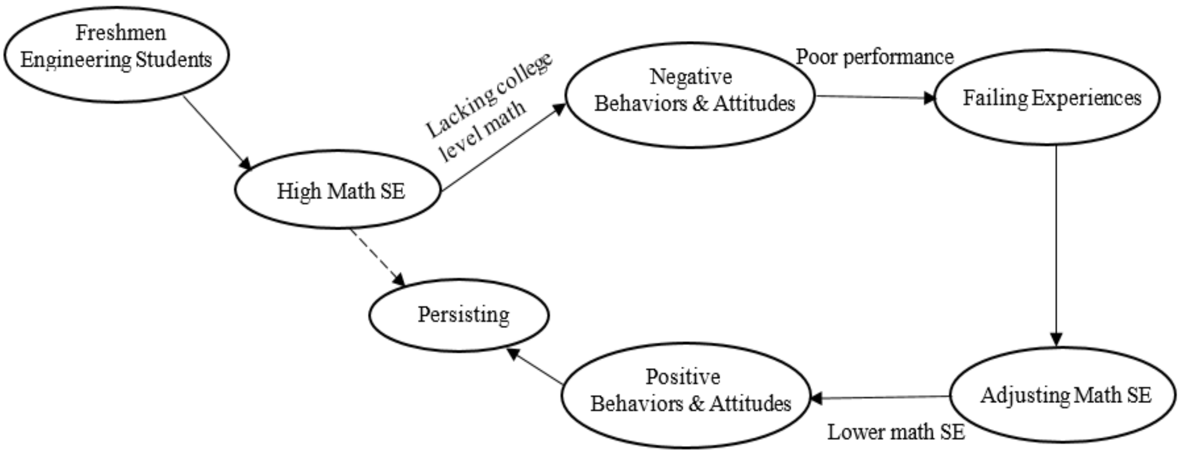

Figure 3. Usual behaviors and attitudes in college mathematics courses for first-year engineering students with high mathematics self-efficacy and poor mathematic preparation as demonstrated by lower mathematics competence.

Note that in Figure 3, "Positive Behaviors and Attitudes" refers to spending extra time, seeking help, and taking struggles as opportunities; "Negative Behaviors and Attitudes" refers to lacking effort, experiencing stress, blaming other factors, and overconfidence.

The relationship between high mathematics self-efficacy and lower mathematics competence suggested by our findings is not accounted for in the current mathematics self-efficacy literature (Bandura, 1986; Hackett & Betz, 1989). These research findings are inconsistent with the most common findings in self-efficacy studies, where students are more likely to be engaged and try to spend more time on activities where they think that they can perform well (Bouffard-Bouchard, 1990; Liem, Lau, & Nie, 2008). Most prior research about self-efficacy is based on this factor alone, and how this factor relates to or affects other behavioral variables such as performance (Vancouver & Kendall, 2006), anxiety (Pajares & Miller, 1994), or academic choices (Schunk, 1991). The mixing phase in our study, which analyzes matches and mismatches of mathematics self-efficacy and competence, helped the researchers to expand current knowledge about self-efficacy. The comparison of mathematics self-efficacy levels with engineering students' mathematics competence showed that there is usually a mismatch between these two factors for first year engineering students, suggesting that the analysis of students' mathematics self-efficacy levels alone is not enough to determine the possible students' behaviors and attitudes in mathematics courses. It may be beneficial for future mathematics self-efficacy research to analyze their participants' mathematics self-efficacy together with their mathematics competence to determine effects of a possible mismatch between these two factors.

Conclusion and Recommendations

Engineering and mathematics educators should be aware that engineering students are likely to have high mathematics self-efficacy beliefs coming from high school, believing they can perform well in college mathematics courses even when they have some deficiencies in their mathematics competence. Mathematics instructors should be prepared to advise and effectively motivate engineering students with a poor mathematics preparation who are not ready to take a calculus course when they start their college education. First year engineering students tend to be overconfident about their mathematics abilities the first time they take a college mathematics course; they tend to procrastinate in class, put little effort trying to improve their mathematics abilities, and blame other factors for their struggles in learning new mathematics topics. The combination of these beliefs, attitudes, and actions may result in them failing their first college mathematics course.

Mathematics instructors should be prepared to inform students about the potential negative behaviors and attitudes related to engineering students with overestimated mathematics self-efficacy beliefs. With this knowledge, these students may reflect on their competence more honestly and could adjust their mathematics self-efficacy beliefs before experiencing a failing grade in their first college mathematics course. If they are advised to align their mathematics self-efficacy beliefs closer to their mathematics competence, they may be more likely to avoid the negative behaviors described above and be more engaged in activities that could help them to improve their mathematics competence and be less likely to fail their first mathematics course in college. Although engineering students will normally use a failing experiences in college mathematics courses to adjust their mathematics self-efficacy beliefs closer to their real mathematics competence level, positively changing their behaviors and attitudes in future mathematics courses, mathematics professors should warn engineering students with poor mathematics preparation about the risk of having a mismatch between high mathematics self-efficacy beliefs with lower mathematics competence during the first weeks of the semester. This knowledge may allow engineering students to avoid failing experiences in their first college year and avoid second thoughts and negative feelings about their abilities to successfully complete all their mathematics courses required by their major. By decreasing the risk of first-year engineering students becoming overconfident about their mathematics competence, engineering educators may also seek ways to reduce the high failing rates in college mathematics courses, having a positive impact in the engineering graduation rates (Seymour & Hewitt, 1997; Suresh, 2006). It may be helpful to have students assess their own mathematics self-efficacy using an instrument like the MSES. We demonstrated validity and reliability for results from the MSES for a population of first-year engineering students.

This research contributes to the mathematics self-efficacy literature by suggesting that high mathematics self-efficacy beliefs that are not supported by a correspondingly high mathematics competence could lead to negative behaviors and attitudes such as overconfidence, lack of effort in mathematics classes, laying blame on other factors for their struggles learning mathematics, and feelings of stress and anxiety when performing mathematics activities, which ultimately lead students to perform poorly and fail their first college mathematics courses.

Despite the negative behaviors and attitudes related to engineering students who show a gap between high mathematics self-efficacy and lower mathematics competence, these students are very likely to keep trying to complete the mathematics courses required by their majors regardless of facing failing experiences because of their high levels of mathematics self-efficacy. Engineering and mathematics educators should try to take advantage of this persistence shown by engineering students by trying to engage them in mathematics related activities. Students with high levels of mathematics self-efficacy are likely to respond positively to a professor's constructive feedback about their mathematics deficiencies, as their mathematics self-efficacy enables them to enjoy working on mathematics related activities but also feel capable of performing well on them.

Limitations

This research should be viewed considering its limited sample, interpretive nature, and the complexity of the theoretical sampling to select the most appropriate participants for the qualitative phase of this research. Firstly, a bigger and more diverse sample would be required to complement the emerging theory with a deeper understanding of the causality of these behaviors and attitudes. Collecting additional data from more students would be especially beneficial for expanding groups that showed some contradictions in student's behaviors. For example, the small sample size (n = 2) of the high mathematics self-efficacy with lower mathematics competence group made it challenging to determine the most likely behaviors and attitudes of participants in this cluster.

Additional data would also support the use of k-means cluster analysis for purposeful sampling of participants, as k-means is appropriate for large sample sizes (Ehlert et al., 2017). The use of k-means cluster analysis with small sample size is a limitation of this study. Secondly, recruiting the right participant that could fit the theoretical sampling specifications was a major challenge due to the lack of response from students to the e-mail invitation. Failing to recruit the right participants to continue developing the emerging theory was the biggest limitation of this research; this issue affected the sample size for groups that needed more participants, particularly those with both low mathematics self-efficacy and competence. Finally, there are more Spring semester (seven) than Fall semester (four) participants in this research. The last qualitative data collection was performed during the Spring 2016 semester, and the last four participants were repeating their mathematics course after failing it the first time. This inability to recruit a participant who had not had a failing experience affected the theoretical sampling. This issue could be addressed in future research by aiming to develop a better understanding of engineering students' behaviors and attitudes prior to having a failure experience.

Future Work

Future research should be conducted to determine the reasons *why* students in different groups are likely to choose certain behaviors in mathematics classes depending on their mathematics self-efficacy level and how well this mathematics self-efficacy is representing their mathematics competence. Additional research with participants from diverse backgrounds should be added to this grounded theory study aiming to expand the emerging theory, and trying to reach theoretical saturation. More data from engineering students in different backgrounds will help to describe students experiences in a richer way (Bryant & Charmaz, 2007), which was not possible to reach during this research due to its time and participant recruitment limitations. Future research aiming to shed light on the possible relationships between mathematics self-efficacy and engineering students' behaviors and attitudes should consider the importance of comparing students' mathematics self-efficacy with their mathematics competence. This research may guide researchers focusing on only self-efficacy measurements rather than analyzing supports for students' beliefs about their abilities. Some students commented on factors outside their control, such as teaching styles and classroom environments, on their academic performance. This indicates a need for future research into the relationships between a mathematics self-efficacy and competence mismatch and student perceptions of locus of control.

Acknowledgements

We wish to thank Dr. Courtney Faber and Catherine McGough, who helped collect interview data and review coded transcripts, and Aubrie Pfirman and Dr. Rachel Anderson for their constant reviewing of the manuscript. We are indebted to Dr. Sandra Linder for generously sharing her expertise on research methods as we designed and conducted this mixed methods study. This research was supported through funding by the Mexican Consejo Nacional de Ciencia y Tecnología.

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Author Information

Gustavo Morán-Soto
Instituto Tecnológico de Durango
Mexico
Contact e-mail: *gmorans@clemson.edu*

Lisa Benson
Clemson University
U.S.A.