The Effect of Using Concrete-Representational-Abstract Sequence in Teaching the Perimeter of Geometric Shapes for Students with Learning Disabilities

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Article Info	Abstract
	Geometry is one of the basic areas of school mathematics education, and it is important for elementary students. However, students with mathematics learning disabilities (MLD) struggle with geometry learning. Research has demonstrated that concrete-representational-abstract (CRA) teaching is an effective practice for students with learning disabilities (LD) and other disabilities. The study aimed to investigate whether the CRA sequence can support students' understanding of calculating the perimeter of geometric shapes and solving mathematical word problems of the perimeter in Najran city in the Kingdom of Saudi Arabia. The study used the quasi-experimental research design for one treatment group of a pre-posttest. First, the perimeter test of geometric shapes, whose validity and reliability was verified, was used, and then the CRA sequence was applied to eight students in the fourth and fifth grades with LD in mathematics. The results showed that the CRA sequence improved the students' efficiency in calculating the perimeter of geometric shapes and solving mathematical word problems on the perimeter. Also, the students maintained the learned skills three weeks post the interventional period.

Introduction

Mathematics is considered one of the basic subjects of great importance in the various stages of education. It contributes greatly to the development of students' mental abilities because of its direct or indirect applications in daily life situations, which earned it a prominent position among study subjects. Also, learning mathematics is not an easy task as it is one of the cumulative fields that require sequential growth within the different stages of growth and requires previous cognitive skills to learn mathematics.

One of the most essential components in the subject of mathematics is the basic concepts of mathematics. The National Council of Teachers of Mathematics (NCTM, 2000) stressed the need for students to learn geometric concepts and identify the properties of geometric shapes because they develop their ability to distinguish between differences and similarities. Also, they are considered a tool to activate the student's mental processes,

so he discovers mathematics and the world and can confront and solve problems (Al-Tahl, 2018).

Geometry is also one of the basic fields of school mathematics education. Students need a basic understanding of measurement and geometry at an early age to support their understanding of complex mathematics later in life (Goldenberg & Clements, 2014). In addition, geometry provides a natural basis for developing reasoning and justification skills (NCTM, 2000), and helps connect facts, elicit results, build the student's personality (Abu Zina, 2010), and provides opportunities to improve cognitive performance, communication processes, and language understanding (Cawley et al., 2009).

Moreover, geometry is one of the ten areas of knowledge related to professions based on science, technology, engineering, and mathematics (STEM) (Carnevale et al., 2011). Geometry is also interwoven with the individual's life and everything that surrounds him. This calls for increased interest in geometry, especially in the early years of education where the focus is on geometric shapes, their properties, and relationships among them, and abstract thinking (Crompton, 2013).

Many students with mathematics difficulties face challenges in learning both types of mathematical concepts, whether numbers (arithmetic, algebra, and numerical analysis) or mathematics of space (geometry). In addition to that, they lag behind their ordinary peers. The reason for students' failure is not due to a lack of effort, but rather due to the difficulty of cognition (Ginsburg, 1997). This category of students is called mathematics difficulties, also known as developmental mathematics difficulties. They are defined as specific and persistent learning disability that affects the development and performance of arithmetic skills (Kucian & von Aster, 2015). It is also one of the most common types of academic difficulties among students in the different stages of education (AL-salahat & Saleem, 2020). Rubisten and Henik (2009) and Schulz et al. (2018) indicated that the prevalence of difficulties in learning mathematics ranged between (5-7%). The manifestations of LD in mathematics vary according to the school stage, and these manifestations differ from one student to another.

In the elementary stage, students with mathematics difficulties show difficulties in distinguishing sizes and shapes and producing geometric shapes (Obeid, 2009). Also, they face difficulties in understanding length, perimeter, area, and size, and converting between units of length, area, and size (Al-Bataineh et al., 2007), and difficulties in perceiving the differences between geometric shapes (Abu Nyan, 2002). In addition, they face difficulties in memorizing and learning mathematical concepts such as the concept of the triangle and its types (Periklidakis, 2003) and the concepts of the quadrilateral (Ma et al, 2015), distinguishing between basic geometric shapes, measurement, and calculating the perimeter and area (Ziadah, 2006). More difficulties of learning mathematics for students with disabilities include learning geometric concepts and weakness in acquiring their concepts (Ibrahim, 2009), and solving mathematical word problems (Zhen, 2009). Further, students also face multiple challenges including organization, problem-solving, work, long-term memory, reading, place value, and arithmetic (Calhoon et al., 2007; Geary et al. 2007; Jitendra et al., 2002; Parmar et al., 1997).

In the field of intervention with students with disabilities including those with LD and MLD, the National

Council of Teachers of Mathematics (NCTM, 2000) recommends that students have an opportunity to develop an understanding of mathematical concepts and procedures through engaging in meaningful mathematics education. The Common Core Standards for Mathematical Practice (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) highlights process standards that prioritize thinking in mathematics and making connections. However, the results of previous research of interventions in mathematics for low-achieving students indicated that teaching students with disabilities focus on teaching computational skills and procedures rather than conceptual knowledge (Bottge, 2001). In addition, the mathematics achievement gap between students with disabilities and chronological age-matched students persists because students with disabilities progress at a much slower rate compared to their normal peers (Bottge, 2001; Cawley & Miller, 1989).

Proceeding from the principle of learning, one of the NCTM Principles and standards for school mathematics states that students should learn mathematics with effective understanding and construction of new information from previous experience and information (NCTM, 2000). The emphasis in mathematics education for students with mathematics difficulties and ordinary students should be conceptual and procedural knowledge. Conceptual knowledge plays a major role in developing a deeper understanding of different mathematical concepts by linking current learning with what exists previously (previous learning) and understanding the relationships and patterns between these different pieces of information (Miller & Hudson, 2007). Whilst procedural knowledge is useful to know mathematical word problem-solving procedures such as step-by-step algorithms, that students learn at school.

To meet the challenges that students with MLD face in learning various academic skills such as reading, writing, and mathematics, teachers often turn to instructional strategies that have been proven successful in research and practice. These strategies or practices are often referred to as evidence-based practices. To date, some interventions or approaches to mathematics have been evaluated or categorized as evidence-based practices for students with LD (Jitendra et al. 2016). Among these evidence-based interventions is CRA sequence. Satsangi et al. (2018) indicated that teaching mathematics to students with LD is in its best form when the gradual transition from concrete to abstract is taken into account. Students with LD are certainly able to achieve high levels of success with complex mathematical concepts, especially when they are supported by visual representations (Marita & Hord, 2017).

Concrete-Representational-Abstract (CRA)

CRA sequence is an evidence-based practice that teachers can use in teaching mathematics sequentially and by following clear and explicit instructions within three phases starting with the concrete stage using manuals, representation through graphics, and abstract using symbols. CRA is of great interest among workers in the field of disability as educational literature shows that CRA is a successful process for teaching students in general education and special education (Watt et al. 2016). It has been proven to be an effective way to help at-risk learners (NMAP, 2008) and one of the recommended practices for use with students with disabilities (Gersten et al., 2009; Powell, 2015). In the field of disability, it was used in teaching mathematics to students with autism

spectrum disorder and developmental disabilities, and was found that CRA is effective in improving the performance of students who participated in studies (Flores et al., 2014; Strozier et al., 2015; Root et al., 2020; Yakubova et al., 2015).

Explicitly and sequentially, the implementation of a CRA model is very important (Agrawal & Morin, 2016; Stroizer et al., 2015). When the teacher teaches within the CRA hierarchy, he must initially model the mathematical concept and provide instructions and appropriate support for students to solve mathematical problems independently. Within the three stages of CRA, students in the first stage learn mathematical concepts and skills through the manipulation of objects. Students in this stage form the meaning of the mathematical concept through the manipulation of objects (Miller & Hudson, 2007), thus, understanding the content of mathematics conceptually (Agrawal & Morin, 2016).

CRA also includes discovery learning strategies that include acting to help students move between conceptual knowledge and procedural knowledge (Sealander et al., 2012). In the next stage, mathematical concepts are presented at a semi-concrete or representational level, in which pictures or graphics are used. Students make their representations of the process and understand the meaning of these representations and the relationships with other operations (Miller et al., 2011).

The abstract level is the last stage of CRA, in which mathematical tasks are completed using numbers only. Students associate previously formed representations with symbols. At this stage, education builds on conceptual understanding and develops procedural knowledge and fluency (Milton et al., 2019). Once students have mastered solving problems with numbers only, the focus in mathematics education is on spontaneity and accuracy, which is the common approach in mathematics education.

Teachers focus on students' mastery of solving problems (e.g., multiplication facts) and then move to accuracy and speed of response. It is necessary and important in the framework of CRA and while working with students that there is close monitoring of progress at each stage so that teachers can ensure that students achieve a level of mastery in each stage before moving on to the next step (Akinoso, 2015). Moving to the next stage before students have mastered the previous stage will lead to student regression and may inhibit conceptual understanding of the target skills (Akinoso, 2015; Fyfe et al., 2014).

The literature deals with a wide range of research and studies that examined teaching mathematics to students using the CRA model. These students developed their conceptual understanding of abstract mathematical concepts through clear, explicit, and sequential instructions included in the CRA stages. CRA was used to teach many mathematical concepts and skills, including the four arithmetic operations (Bouck et al., 2018; Flores, 2010; Flores, & Hinton, 2019; Kim, 2015; Mancl et al., 2012; Miller & Mercer, 1993; Miller & Kaffar, 2011; Sealander et al., 2012;). CRA was used to teach fractions (Bouck et al., 2017; Butler et al., 2003; Flores et al., 2020; Jordan et al., 1999; Lemonidis et al., 2020; Misquitta, 2011; Morano et al., 2020), perimeter and area (Cass et al., 2003; Indriani,2019; Satsangi & Bouck, 2015;), vocabulary problems and place value (Doabler & Fien, 2013), logical-mathematical thinking (Novaliyosi, 2020), rounding, regrouping, and equivalent fractions

(Hinton, Flores, 2019), Algebra (Scheuermann, 2009; Strickland & Maccini, 2013; Witzel, 2005; Witzel et al., 2003), and place value and geometry (Fuchs et al., 2007).

Statement of Problem

Concerning the recommendations of the National Council of Teachers of Mathematics, it is necessary to focus more on geometry at all educational levels and to consider it one of the most prominent standards of mathematics in the twentieth century due to its connection with the structure of the individual and his daily life and other mathematical and scientific topics (NCTM, 2000). In addition, geometry represents the largest part of concrete mathematics that is easy for the student to learn especially if the appropriate educational methods and manuals are available to understand and master. This study aims to reveal how CRA sequence is used to help students with MLD in finding the perimeter of geometric shapes. The CRA sequence is implemented on eight students with MLD from the fourth and fifth grades using the quasi-experimental research design for one treatment group of a pre-posttest in Saudi Arabia. Students in the fourth grade begin to learn the perimeter of geometric shapes and solve mathematical problems on the perimeter. Geometric concepts and skills such as perimeter and area are one of the goals included in the individual educational plans for the students of the study sample.

Also, the application date of the study coincides with the date of presenting the topic of the perimeter of geometric shapes to all students. In turn, this helps students with mathematics difficulties to learn the concepts and skills related to the perimeter of geometric shapes in a better way, which has a significant impact on students with difficulties learning mathematics in catching up with their ordinary peers. In this study, the three phases of concrete, representational, and abstract were combined in one session. Thus, it is similar to the study of Zhang et al. (2021) and differs from the rest of previous studies.

Research Questions

This study will attempt to answer the following questions:
1. Is there an effect of using concrete, representational, and abstract sequence on improving students' performance in calculating the perimeter of geometric shapes, solving mathematical word problems in the pre and post-tests?
2. To what extent does the performance of students with LD differ on the delayed test in calculating the perimeter of geometric shapes after three weeks of application?

Method

Research Design

The quasi-experimental research design for one treatment group of a pre-post test was used in this study. It is used to calculate the effect size between the independent and dependent variables.

Participants

Eight students participated in the study, four of them in the fourth grade and four in the fifth grade. Their ages ranged between 10-11 years. They were chosen purposefully due to the availability of the study sample, students with MLD, a LD program at school, and the cooperation of the school administration with the researcher. The criteria for participation in the study were as follows:

(1) the consent of parent and student to participate,

(2) they are among the students diagnosed with difficulties in learning mathematics through formal and informal tests,

(3) the mathematics teacher's observations that show weaknesses in students' performance that are consistent with the results of formal and informal tests.

Settings

This study was conducted in a primary school (from the first to the sixth grade) in the city of Najran, located in the south of the Kingdom of Saudi Arabia. Students with LD receive special education services in the resource room, which is considered an educational alternative approved in the Kingdom of Saudi Arabia. They are provided with support in mathematics at a rate of two to three lessons per week and in a period ranging between 30-45 minutes. The LD teacher gave instructions to the students in groups of four students.

The students were taught at a rectangular table, and the teacher sat close to them, taking into account the ease of following up with the students during their work. The teacher used the manuals for the study, which are a set of geometric tapes and figures of numbers. It should be noted that the LD teacher who carried out the study is a qualified teacher to work with students with LD. He holds a bachelor's degree in special education, LD track, with more than 7 years of experience. In addition, he is a graduate student in the master's program in special education, LD track.

Materials

The study included multiple materials suitable for CRA stages. In the concrete stage, geometric strips were used, which are plastic tapes of different colors and lengths, perforated at both ends and in the middle, and clips to connect pieces. For example, if the student is asked to make a geometric shape, let it be a quadrilateral, he chooses four pieces and four clips and then connects them to form the shape.

If the desired shape is a regular polygon, he chooses four pieces of equal length. In the second stage, they used papers, pens, and a ruler to draw the desired shape, taking into account not to focus too much on the drawing accuracy as the main goal of this stage is to embody the handmade shape by drawing and more deeply in understanding the mathematical concept. In the last stage, papers and pens were used to solve the mathematical problem with symbols.

CRA Intervention

The intervention material was prepared in teaching the perimeter of geometric shapes based on the three steps of (the CRA) sequence. To facilitate the transition between the three levels, concrete, representational and abstract, the intervention sessions in this study were prepared by merging these three levels in one session. Students were taught the perimeter of geometric shapes using geometric tapes at the concrete level, drawing the geometric shape at the representational level, and finally the strategy of remembering and numbers at the abstract level by converting the drawing into an equation to calculate perimeter, and a detailed presentation of the method of intervention through (CRA) follows.

Stage One: Concrete

This stage is called the "work" stage where students with MLD physically manipulate objects to solve a mathematical problem through the use of 3D artifacts to help them learn new concepts (Miller & Kaffar, 2011). With this in mind, students will be able to move and manipulate 3D objects to represent their thinking. For example, the student uses geometric tapes to form a triangle with different sides, or equilateral, and represents the length of each side using numbers (see Table 1). In addition, the use of manual methods increases the number of sensory inputs that the student uses while learning a new concept, which improves the student's chances of remembering the procedural steps necessary to solve a certain problem (Witzel, 2005), for example, the colors and lengths of geometric bands (see Table 1).
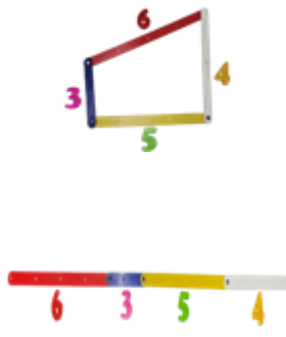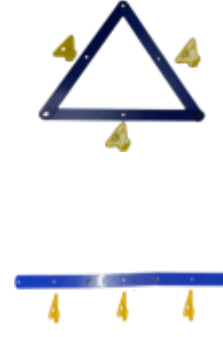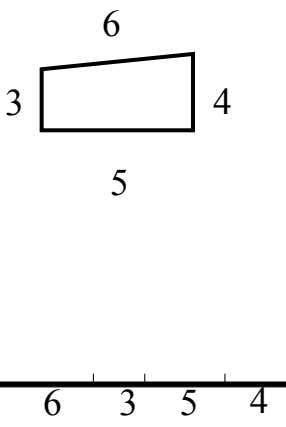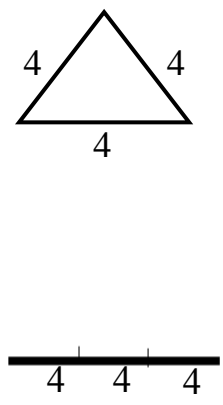
Stage Two: Representational

This stage is called the "vision" stage and involves using pictures to represent things to solve a mathematical problem. This stage requires students to do a simple drawing of the concrete things they used in the first stage, so students' mastery of the first stage is a prerequisite for moving to the second stage. At this stage, the teacher must clarify the relationship between drawing and concrete things and must provide many training examples for students to get them to work independently. For example, draw (triangle, straight segment) (Table 1). Also, re-training in visualizing the concrete stage with a simple drawing would help students understand the mathematical skill/concept.

Stage Three: Abstract

This stage is called "symbolic", begins after the student demonstrates a thorough understanding of the representational level, and involves using only numbers and symbols to solve arithmetic problems. Students no longer rely on manual methods or graphics to solve problems (Witzel et al., 2008). At this stage, students only use mathematical strategies to solve problems (Agrawal & Morin, 2016; Doabler & Fien, 2013). For example, speak in your language how to find the perimeter of a regular/irregular polygon, represent the perimeter of a regular/irregular polygon with an equation (see Table 1). The problem can also be solved using abstract symbolic notation, which involves memorizing mathematical procedures and continues until the student learns

the procedure or concept automatically (Flores, 2009; Witzel, 2005; Witzel et al., 2008).

Table 1. An Example of CRA Sequence

Steps	Procedures	Geometric shapes	
		Irregular	Regular
Concrete "work stage"	- Model the desired shape using geometric tapes and clips. -Put the number that represents the length of each side of the shape. -Loosen one of the clips to make a straight segment. - Add all the numbers to calculate the perimeter of the desired shape.	 6+3+5+4=18	 4+4+4=12
Representational "vision stage"	-Draw the desired shape. - Write the length of each side of the desired shape on the drawing. - Draw a line segment and divide it by the number of sides of the shape and represent the numbers on it. (Example: For a triangle, a straight line segment is divided into three segments). - Add all the numbers to calculate the perimeter of the desired shape	 6+3+5+4=18	 4+4+4=12
Abstract "symbolic stage"	- Express using your own language how to find the desired shape. - Represent the perimeter of the desired shape with an equation. - Calculate the perimeter of the desired shape using numbers only.	Perimeter = sum of the lengths of sides 6+3+5+4=18	Perimeter = sum of the lengths of sides 4+4+4=12 Perimeter = number of sides × side length 3×4=12

Instrument

To measure the level of students' performance in calculating the perimeter of geometric shapes, the researcher prepared a test for the perimeter of geometric shapes. The test was based on mathematics curricula and the diagnostic tests approved by the Department of Special Education in Najran city in the Kingdom of Saudi Arabia. The test for the perimeter of geometric shapes consisted of a set of skills such as calculating the perimeter of irregular polygons, regular polygons, and solving mathematical word problems on the perimeter.

To verify the validity of the test content, it was reviewed by (13) experts in the field of LD and mathematics. Based on the experts' observations, the items that were agreed upon by (11) out of (13) experts were retained, i.e. with an agreement rate of (85%). Thus, the final version of the perimeter test of geometric shapes consisted of 12 questions: (4) questions for irregular geometric shapes, (4) questions for regular geometric shapes, (4) mathematical word problems calculating the perimeter. Each question has 3 marks, so the highest score is 36 and the lowest is zero. The reliability of the test was verified using Coder Richardson 21. The reliability coefficient on the overall test score was (89%).

Results

To answer the study questions, the means and standard deviations of the students' scores were calculated on the pre, post, and delayed tests as shown in Table 2.

Table 2. Means and Standard Deviations of the Students' Scores

Perimeter of geometric shapes	Test	N	Mean	Std. Deviation
Irregular shapes		8	5.25	1.035
Regular shapes	Pretest	8	2.38	.518
Word problem-solving		8	1.75	.707
Overall		8	9.38	1.408
Irregular shapes		8	12.00	.000
Regular shapes	Posttest	8	11.75	.463
Word problem-solving		8	9.00	1.309
Overall		8	32.75	1.282
Irregular shapes		8	12.00	.000
Regular shapes	Delayed test	8	11.50	.535
Word problem-solving		8	8.63	1.408
Overall		8	32.13	1.642

It is evident from Table 2 that the average score of students in the pretest increased from (9.38 to 32.75) in the posttest. To illustrate, the average score of students in the perimeter skills (irregular shapes, regular shapes, word problem-solving) increased from (5.25, 2.38, 1.75) in the pretest to (12.00, 11.75, 9.00) in the posttest respectively.

To answer the first research question, which aims to identify the effect of using (CRA) on improving students' performance in calculating the perimeter of geometric shapes in the pre and post-tests, the Wilcoxon test for non-independent samples was used to calculate the significance of the differences between the students' scores in the pre and posttests as displayed in Table 3.

Table 3. Wilcoxon Test for Students' Scores in the Pre and Posttests

Perimeter of geometric shapes	Negative Ranks		Positive Ranks		Test Statistics		
	Mean Rank	Sum of Ranks	Mean Rank	Sum of Ranks	Z	p	r
Irregular shapes	.00	.00	4.5	36.00	-2.539	.011	.63
Regular shapes	.00	.00	4.5	36.00	-2.565	.010	.64
Word problem-solving	.00	.00	4.5	36.00	-2.536	.011	.63
Overall	.00	.00	4.5	36.00	-2.527	.012	.63

Table 3 shows that the difference between the ranks of students' scores in the two tests, the pre, and post, revealed statistically significant differences on the test after applying the (CRA) instructional approach. The level of statistical significance was (.012) which is less than (.05), and the differences were in favor of the students in the posttest application. It is also clear that there were differences in favor of students in the posttest on the skills (irregular shapes, regular shapes, word problem-solving) respectively. The effect size was (.63, .64, .63) and at a moderate impact level on the three skills.

The second research question aims to identify whether the use of (CRA) has an impact on the retention of the learning effect in the post and delayed tests. The Wilcoxon test for non-independent samples was used to calculate the significance of the differences between the grades of students' scores in the post and delayed tests as depicted in Table 4.

Table 4. Wilcoxon Test for Students' Scores in the Pre and Delayed Tests

Perimeter of geometric shapes	Negative Ranks		Positive Ranks		Test Statistics	
	Mean Rank	Sum of Ranks	Mean Rank	Sum of Ranks	Z	p
Irregular shapes	.00	.00	.00	.00	.000	1.000
Regular shapes	1.5	3.00	.00	.00	-1.414	.157
Word problem-solving	3	6.00	.00	.00	-1.732	.083
Overall	2.5	10.00	.00	.00	-1.890	.059

It is clear from Table 4 that there were statistically significant differences at (0.05) on the test of the perimeter of geometric shapes as a whole and the three skills (irregular shapes, regular shapes, word problem-solving) after applying (CRA) instructional approach between the post and delayed tests. The level of statistical significance is greater than (0.05), and this indicates the retention of the learning effect.

Discussion

The study aimed to examine the effect of using CRA sequence on students' performance with MLD in calculating the perimeter of geometric shapes and solving mathematical word problems on the perimeter of geometric shapes. The findings indicated an improvement in the performance of students with MLD using the CRA sequence. The students also maintained the learned skills three weeks after the end of the treatment. The error patterns observed before the intervention did not persist and were not evident after the clear and explicit instructions of the CRA sequence. This sequence improved the students' abilities to calculate the perimeter of regular and irregular geometric shapes. The students showed an improvement in solving mathematical word problems on the perimeter of geometric shapes. The findings obtained from this study support the sequential approach (CRA) as an effective approach to teaching mathematical concepts to students with LD (Bouck et al., 2018; Flores et al., 2020; Hinton & Flores, 2019; Kim, 2015; Mancl et al., 2012; Miller & Mercer, 1993; Milton et al., 2019; Morano et al., 2020; Novaliyosi, 2020; Strickland & Maccini, 2013; Witzel, 2005; Zhang et al., 2021). It provides further evidence that CRA sequence has improved the performance of students with LD. The findings of this study are in agreement with those of previous studies (Indriani, 2019; Cass et al., 2003) which indicated that the use of CRA sequence led to an improvement in mathematics learning about the perimeter and area of geometric shapes. Students also preserved these skills.

The improvement in the performance of students with MLD can also be attributed to the method used in the intervention process in this study. It began with a focus on building conceptual understanding and then procedural knowledge. There was an increase in the average scores of students between the pre and post-test. The effect size was moderate. This finding is consistent with what was indicated by Milton et al. (2019) in that conceptual and procedural knowledge is useful in developing understanding and procedural knowledge to solve mathematical problems respectively and that conceptual knowledge is important and strengthens procedural knowledge, and that both are necessary components for mathematical competence (Rittle - Johnson et al., 2001).

By looking closely at the students' results, the effect of using the sequence (CRA) on their ability to calculate the perimeter of geometric shapes was evident in the comparison between the students' scores on the pre and post-test. This effect also appeared after five sessions of intervention, through the examples that the students solved after the completion of each session. The use of the (CRA) sequence led to an immediate improvement in calculating the perimeter of geometric shapes. For example, when presenting a geometric shape to students on paper to calculate its perimeter, they may not understand the meaning of the number that represents the length of each side of the regular and irregular geometric shape. This could be the reason behind the difficulty the students faced in the pretest. These difficulties were overcome when students applied the three steps of the sequence (CRA) by designing intensive sessions with three combined levels in which the geometric shape was formed using manuals (geometric tapes). Then, the shapes were represented by a simple drawing of the geometric shape using paper and pen and finally writing the perimeter of a regular/irregular polygon with an equation.

In addition, the increase in training opportunities provided by the teacher during the intervention stage and on

the three stages of the CRA sequence and not moving to the next stage until mastering the previous stage helped improve the students' conceptual and procedural knowledge. This can be evident in (a) the results of the daily assessment during the intervention sessions, (b) the improvement in the students' scores with MLD in the posttest, and (c) the retention of the learned skills after three weeks of intervention. Fyfe et al. (2014) confirmed that moving to the next stage before students have mastered the previous stage will lead to a decline in students' scores and may prevent conceptual understanding. For example, the teacher returned to the first stage (concrete) with one of the students, modeled the required shape in front of the student, and then asked him to model the shape while instructing him to focus on the colors of the geometric bars that represent the shape. This helped the student understand the concept. This is consistent with what was indicated by Witzel (2005) that the use of manual methods increases the number of sensory inputs used by the student while learning the new concept, which improves his chances of remembering the procedural steps necessary to solve a problem.

The improvement can also be attributed to the immediate feedback provided by the teacher to students during intervention sessions and at each stage (CRA), and then work to gradually reduce direct intervention during the training, to build the students' ability to work independently and self-reliance. In addition, this improvement can be attributed to the flexible planning adopted in the current study, that is, the transition to the next skill is based on the student's mastery of the previous skill. This is in line with what was indicated by the LD teacher while working with students that there was a positive interaction with students during the explanation of CRA sequential lessons. Correcting the wrong individual performance and enhancing the correct performance helped in the development of their cognitive processes.

Conclusion

The purpose of the current study was to teach students with difficulties learning mathematics to calculate the perimeter of geometric shapes and solve mathematical word problems on perimeter. There were statistically significant differences in the mean scores of students on the pre and post-tests. The intervention was successful as an intensive intervention with a small number of students (3-4) students per session. This success was achieved by the LD teacher through a short intervention that included accessible materials and required little professional development. The findings of the current study are also important in that they demonstrated the efficacy of combining the three stages of CRA. Therefore, this is a promising intervention and future research is needed to replicate and confirm these findings.

Acknowledgment

References

Abu Nyan, I. (2002). *Learning Difficulties: Teaching Methods and Cognitive Strategies*. Kingdom of Saudi

Arabia, Riyadh: Academy of Special Education.

Abu Zina, F. (2010). *School Mathematics Curricula and Teaching*-1st Edition. Dar Wael for Publishing and Distribution, Amman.

Agrawal, J., & Morin, L. L. (2016). Evidence-based practices: applications of concrete representational abstract framework across math concepts for students with mathematics disabilities. *Learning Disabilities Research & Practice (Wiley- Blackwell), 31*(1), 34-44. doi:10.1111/ldrp.12093

Akinoso, S. O. (2015). Teaching mathematics in a volatile, uncertain, complex and ambiguous (vuca) world: the use of concrete – representational - abstract instructional strategy. *Journal of The International Society for Teacher Education, 19*(1), 97-107.

Al-Bataineh, O., Al-Khattaba, A., Al-Sabayla, O., & Al-Rashdan, M. (2007). *Learning Disabilities: Theory and Practice*. Amman, Jordan, Dar Al Masirah.

Al-salahat, M & Saleem, S. (2020). Effect of Model Drawing Strategy for Fraction Word Problem Solving for Students with Learning Disabilities. *Dirasat: Educational Sciences, 47*(4), 485-502.

Al-Tahl, A. R. H. (2018). *The effect of Gerleach and Ely's model on the acquisition of geometry concepts for sixth grade female students in Jordan and their inclinations towards learning mathematics*. Published master's thesis. College of Graduate Studies, University of Jordan, Jordan.

Bottge, B. A. (2001). Reconceptualizing mathematics problem solving for low-achieving students. *Remedial and Special Education, 22*, 102–112. doi:10.1177/074193250102200204

Bouck, E., Bassette, L., Shurr, J., Park, J., Kerr, J., & Whorley, A. (2017). Teaching Equivalent Fractions to Secondary Students with Disabilities via the Virtual–Representational–Abstract Instructional Sequence. *Journal of Special Education Technology, 32*, 220-231.

Bouck, E., Satsangi, R., & Park, J. (2018). The Concrete-Representational–Abstract Approach for Students with Learning Disabilities: An Evidence-Based Practice Synthesis. *Remedial and Special Education, 39*, 211-228.

Butler, F. M., Miller, S. P., Crehan, K., Babbitt, B., & Pierce, T. (2003). Fraction instruction for students with mathematics disabilities: Comparing two teaching sequences. *Learning Disabilities Research & Practice, 18(2),* 99-111.

Calhoon, M. B., Emerson, R. W., Flores, M., & Houchins, D. E. (2007). Computational fluency performance profile of high school students with mathematics disabilities. *Remedial and Special Education, 28,* 292–303.

Carnevale, A. P., Smith, N., & Melton, M. (2011). *STEM: Science, technology, engineering, mathematics*. Georgetown University Center on Education and the Workforce.

Cass, M., Cates, D., Smith, M., & Jackson, C.W. (2003). Effects of Manipulative Instruction on Solving Area and Perimeter Problems by Students with Learning Disabilities. *Learning Disabilities Research and Practice, 18*, 112-120.

Cawley, J. F., & Miller, J. H. (1989). Cross-sectional comparisons of the mathematical performance of children with learning disabilities: Are we on the right track toward comprehensive programming? *Journal of Learning Disabilities, 22*, 250–259. doi:10.1177/002221948902200409

Cawley, J. F., Foley, T. E., & Hayes, A. M. (2009). Geometry and measurement: A discussion of status and content options for elementary school students with learning disabilities. *Learning Disabilities: A*

Contemporary Journal, 7(1), 21–42.

Crompton, H. (2013). *Coming to understand angle and angle measure: a design-based research curriculum study using context-aware ubiquitous learning*. Unpublished PhD dissertation, University of North Carolina at Chapel Hill.

Doabler, C. T., Fien, H. (2013). Explicit mathematics instruction: What teachers can do for teaching students with mathematics difficulties? *Intervention for School and Clinic, 48*, 276–285.

Flores, M. M. (2009). Teaching subtracting with regrouping to students experiencing difficulty in mathematics. *Preventing School Failure, 53* (3), 145–152. doi: 10.3200/PSFL.53.3.145-152.

Flores, M. M., Hinton, V. M., Strozier, S. D., & Terry, S. L. (2014). Using the concrete representational-abstract sequence and the strategic instruction model to teach computation to students with autism spectrum disorders and developmental disabilities. *Education and Training in Autism and Developmental Disabilities, 49*, 547–554.

Flores, M.M. (2010). Using the Concrete-Representational-Abstract Sequence to Teach Subtraction with Regrouping to Students at Risk for Failure. *Remedial and Special Education, 31*, 195-207.

Flores, M.M., & Hinton, M.M. (2019). Improvement in Elementary Students' Multiplication Skills and Understanding after Learning through the Combination of the Concrete-Representational-Abstract Sequence and Strategic Instruction. *Education and Treatment of Children, 42*, 73-99.

Flores, M.M., Hinton, V., & Meyer, J.M. (2020). Teaching Fraction Concepts Using the Concrete-Representational-Abstract Sequence. *Remedial and Special Education, 41*, 165-175.

Fuchs, L. S., Fuchs, D., & Hollenbeck, K. N. (2007). Extending responsiveness to intervention to mathematics at first and third grades. *Learning Disabilities Research and Practice, 22*(1), 13-14

Fyfe, E. R., McNeil, N. M., Son, J. Y., & Goldstone, R. L. (2014). Concreteness fading in mathematics and science instruction: a systematic review. *Educational Psychology Review, 26*, 9-25.

Geary, D. C., Hoard, M. K., Byrd-Craven, J., Nugent, L., & Numtee, C. (2007). Cognitive mechanisms underlying achievement deficits in children with mathematics learning disability. *Child Development, 78*, 1343–1359. doi:10.111/ j.1467-8624.2007.01069.x

Gersten, R. M., Chard, D., Jayanthi, M., Baker, S. K., Morphy, P., Flojo, J. (2009). Mathematics instruction for students with learning disabilities: A meta-analysis of instructional components. *Review of Educational Research, 79*, 1202–1242. doi:10.3102/0034654309334431

Ginsburg, P. H. (1997). Mathematics learning disabilities: A view from developmental psychology. *Journal of Learning Disabilities, 30*(1), 20–33.

Goldenberg, E. P., & Clements, D. H. (2014). Why geometry and measurement? In B. Dougherty & R. Zbiek (Eds.), *Developing essential understanding of geometry and measurement for teaching mathematics in pre-kindergarten–grade 2* (pp. 1–2). National Council of Teachers of Mathematics.

Hinton, V. M., & Flores, M. M. (2019). The effects of the concrete-representational-abstract sequence for students at risk for mathematics failure. *Journal of Behavioral Education*, *28*(4), 493-516.

Ibrahim, M. A. (2009). *Classroom questions as an introduction to teaching arithmetic for people with learning disabilities*. Book World, Cairo.

Indriani, L. (2019). The Implementation of Concrete-Representational-Abstract (CRA) Approach to Improve Mathematical Learning about perimeter and Area Plane in Students Grades IV SD negeri 2 sempor in

Academic Year 2018/2019. Thesis, Teacher Training and Education Faculty, Universitas Sebelas Maret, Surakarta

Jitendra, A. K., Nelson, G., Pulles, S. M., Kiss, A. J., & Houseworth, J. (2016). Is mathematical representation of problems an evidence-based strategy for students with mathematics difficulties? *Exceptional Children, 83*(1), 8-25.

Jitendra, A., DiPipi, C. M., & Perron-Jones, N. (2002). An exploratory study of schema-based word-problem-solving instruction for middle school students with learning disabilities. *The Journal of Special Education, 36*, 23–38.

Jordan, L., Miller, M. D., & Mercer, C. D. (1999). The effects of concrete to semi-concrete to abstract instruction in the acquisition and retention of fraction concepts and skills. *Learning Disabilities: A Multidisciplinary Journal, 9,* 115-122.

Kim, A. (2015). Effects of using the Concrete-Representational-Abstract Sequence with Mnemonic Strategy Instruction to teach Subtraction with Regrouping to Students with Learning Disabilities. *The Journal of Elementary Education 28*(4), 267–292.

Kucian, K., & von Aster, M. (2015). Developmental dyscalculia. *European Journal of Pediatrics, 174*, 1-13. doi:10.1007/s00431-014-2455-7

Lemonidis, C., Anastasiou, D., & Iliadou, T. (2020). Effects of Concrete-Representational-Abstract instruction on fractions among low-achieving sixth-grade students. *Educational Journal of the University of Patras UNESCO Chair, 7*(2).

Ma, H.L.; Lee, D.C.; Lin, S.H.; Wu, D.B. A study of van Hiele of geometric thinking among 1st through 6th graders. *Eurasia J. Math. Sci. Technol. Educ.* 2015, *11*, 1181–1196

Mancl, D. B., Miller, S. P., Kennedy, M. (2012). Using the concrete-representational-abstract sequence with integrated strategy instruction to teach subtraction with regrouping to students with learning disabilities. *Learning Disabilities Research & Practice, 27*, 152–166.

Marita, S., & Hord, C. (2017). Review of mathematics interventions for secondary students with learning disabilities. *Learning Disability Quarterly, 40*, 29-40.

Miller, S. P., & Kaffar, B. J. (2011). Developing addition with regrouping competence among second grade students with mathematics difficulties. *Investigations in Mathematics Learning, 4,* 24-49.

Miller, S. P., Stringfellow, J. L., Kaffar, B. J., Ferreira, D., & Mancl, D. B. (2011). Developing Computation Competence among Students Who Struggle with Mathematics. *Teaching Exceptional Children, 44*(2), 38-46. https://doi.org/10.1177/004005991104400204

Miller, S.P. and Hudson, P.J., (2007). Using Evidence-Based Practices to Build Mathematics Competence Related to Conceptual, Procedural, and Declarative Knowledge. *Learning Disabilities Practice, 22(*1), 47-57.

Miller, S.P., & Mercer, C.D. (1993). Using data to learn about concrete-representational-abstract instruction for students with math disabilities. *Learning Disabilities Research and Practice, 8,* 89-96

Milton, J. H., Flores, M. M., Moore, A. J., Taylor, J. J., & Burton, M. E. (2019). Using the Concrete–Representational–Abstract Sequence to Teach Conceptual Understanding of Basic Multiplication and Division. *Learning Disability Quarterly, 42*(1), 32–45. https://doi.org/10.1177/0731948718790089

Misquitta, R. (2011). A review of the literature: Fraction instruction for struggling learners in

mathematics. *Learning Disabilities Research & Practice, 26(2),* 109-119.

Morano, S., Flores, M.M., Hinton, V., & Meyer, J. (2020). A Comparison of Concrete-Representational-Abstract and Concrete-Representational-Abstract-Integrated Fraction Interventions for Students with Disabilities. *Exceptionality, 28*, 77 - 91.

National Council of Teachers of Mathematics (NCTM). (2000). *Principles and standards for school mathematics*. Reston, VA: Author

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common core state standards for mathematics*. Washington, DC: Author. Retrieved from http://www.corestandards.org

National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel*. Washington DC: U.S. Department of Education.

Novaliyosi (2020). Perkembangan Kemampuan Berpikir Logis Matematis Melalui Pendekatan Cra (Concrete-Representational-Abstract) Disertai Penilaian Portofolio (Penelitian Quasi Eksperimen dengan Desain Time Series).

Obeid, Magda El-Sayed. (2009). *Learning Disabilities and how to deal with them*. Amman, Dar Safa.

Parmar, R. S., & Cawley, J. F. (1997). Preparing teachers to teach mathematics to students with learning disabilities. Journal of Learning Disabilities, 30, 188–197.

Periklidakis, G. (2003). *Learning difficulties in Mathematics in primary school children with normal intelligence-dyscalculia (Diagnosis-Treatment)*. University of Crete, Rethimno, Greece.

Powell, S. R. (2015). Connecting Evidence-Based Practice With Implementation Opportunities in Special Education Mathematics Preparation. Intervention in School and Clinic, 51(2), 90–96.

Rittle - Johnson, B., Siegler, R. S., & Alibali, M. W. (2001). Developing conceptual understanding and procedural skill in mathematics: An iterative process. *Journal of Educational Psychology, 93,* 346 – 362.

Root, J. R., Cox, S. K., Gilley, D., & Wade, T. (2021). Using a virtual-representational-abstract integrated framework to teach multiplicative problem solving to middle school students with developmental disabilities. *Journal of Autism and Developmental Disorders*, *51*(7), 2284-2296.

Rubinsten, O. & Henik, A. (2009). Developmental dyscalculia: heterogeneity might not mean different mechanisms. *Trends in Cognitive Science 13*(2), 92–99.

Satsangi, R., Bouck, E. C. (2015). Using virtual manipulative instruction to teach the concepts of area and perimeter to secondary students with learning disabilities. *Learning Disability Quarterly, 38*, 175–186.

Satsangi, R., Hammer, R., & Hogan, C. D. (2018). Studying Virtual Manipulatives Paired With Explicit Instruction to Teach Algebraic Equations to Students With Learning Disabilities. *Learning Disability Quarterly, 41*(4), 227–242. https://doi.org/10.1177/0731948718769248

Scheuermann, A. M., Deshler, D. D., & Schumaker, J. B. (2009). The effects of the explicit inquiry routine on the performance of students with learning disabilities on one-variable equations. *Learning Disability Quarterly, 32*(2), 103–120.

Schulz, F., Wyschkon, A., Gallit, F., Poltz, N., Moraske, S., Kucian, K. et al. (2018). Rechenprobleme bei Grundschulkindern: Persistenz und Schulerfolg nach fünf Jahren. *Lernen und Lernstörungen, 7*(2), 67–80.

Sealander, K. A., Johnson, G.R., Lockwood, A. B., & Medina, C. M. (2012). Concrete-semiconcrete-abstract

(CSA) instruction: A decision rule for improving instructional efficacy. *Assessment for Effective Intervention, 30,* 53-65.

Strickland, T. K., & Maccini, P. (2013). The effects of the concrete– representational– abstract integration strategy on the ability of students with learning disabilities to multiply linear expressions within area problems. *Remedial and Special Education, 34*(3), 142–153. doi:10.1177/0741932512441712

Stroizer, S., Hinton, V., Flores, M., & Terry, L. (2015). An investigation of the effects of CRA instruction and students with autism spectrum disorder. *Education and Training in Autism and Developmental Disabilities, 50,* 223–236.

Watt, S., Watkins, J., & Abbitt, J. (2016). Teaching algebra to students with learning disabilities: Where have we come and where should we go? *Journal of Learning Disabilities, 49*(4), 437-447.

Witzel, B. S. (2005). Using CRA to teach algebra to students with math learning disabilities in inclusive settings. *Learning Disabilities: A Contemporary Journal, 3*(2), 49-60.

Witzel, B. S., Mercer, C. D., & Miller, M. D. (2003). Teaching algebra to students with learning difficulties: An investigation of an explicit instruction model. *Learning Disabilities Research & Practice, 18*(2), 121–131. https://doi.org/10.1111/1540-5826.00068

Witzel, B. S., Riccomini, P. J., & Schneider, E. (2008). Implementing CRA with secondary students with learning disabilities in mathematics. *Intervention in School and Clinic, 43*, 270-276.

Yakubova, G., Hughes, E. M., & Hornberger, E. (2015). Video-Based Intervention in Teaching Fraction Problem-Solving to Students with Autism Spectrum Disorder. *Journal of autism and developmental disorders*, *45*(9), 2865–2875. https://doi.org/10.1007/s10803-015-2449-y

Zhang, S., Yu, S., Xiao, J., Liu, Y., & Jiang, T. (2021). The Effects of Concrete-Representational-Abstract Sequence Instruction on Fractions for Chinese Elementary Students with Mathematics Learning Disabilities. *International Journal of Science and Mathematics Education*, 1-18.

Zheng, X. (2009). *Working memory components as predictors of children's mathematical word problem solving processes*. Ph.D. dissertation, University of California, Riverside, United States, California. Retrieved December 4, 2009, from Dissertations & Theses: Full Text. (Publication No. AAT 3374426)

Ziadah, K. (2006). *Difficulties in learning mathematics: dyscalculia*. Cairo, Itrak for printing and publishing.

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