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To cite this article:

Bundock, K., Shumway, J. F., Burnside, M., & King, J. (2023). Teachers' actions and students' engagement behaviors during number system knowledge discussions: Implications for enhancing active engagement. *International Journal of Education in Mathematics, Science, and Technology (IJEMST), 11*(2), 506-526. https://doi.org/10.46328/ijemst.2576

International Journal of Education in Mathematics, Science and Technology

2023, Vol. 11, No. 2, 506-526 https://doi.org/10.46328/ijemst.2576

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Article Info

Article History

Received:
05 May 2022

Accepted:
09 November 2022

Keywords

Student engagement
Number system knowledge
Number sense
Math achievement
Teacher practices

Abstract

A critical challenge for elementary mathematics teachers is meeting the learning needs of all students, especially when there is variability in students' number knowledge. Because young students' number system knowledge (NSK) contributes to future success in mathematics, NSK activities must be designed to engage all students, particularly students who have struggled to engage in mathematics, to help students build early number knowledge. In this manuscript, we describe a quick images number sense intervention implemented across five second-grade teachers and 75 students, and examine outcomes related to teacher actions and student engagement behaviors. The results of our mixed-methods study indicate that the intervention provided multiple ways for students of varying abilities to engage with and learn the content. Our results provide practical implications for teachers, and prompt questions to evaluate in future research.

Introduction

Students enter school with considerable variability in early number knowledge (Geary et al., 2013), which persists through the upper grades and affects students' mathematics achievement (Morgan et al., 2009). Variability in number knowledge poses a challenge for elementary teachers to meet the learning needs of all students. One component of number knowledge, *number system knowledge,* contributes to young students' future success in mathematics (Geary et al., 2013; Geary & vanMarle, 2016). Number system knowledge (NSK) is the ability to relate quantities to their respective numeral representations, understand relations among those numbers, and use that knowledge to operate on quantities (Geary et al., 2017). Activities promoting connections between related visual quantities, numerals and equations provide students opportunities to develop NSK (Shumway et al., 2020), for instance, seeing an image of 12 dots and naming and combining parts of that total such as *ten and two are twelve* (10 + 2 = 12) or *six and six make twelve* (6 + 6 = 12). While these activities generally provide students with opportunities to interact with NSK, they must be designed to provide access to all students in order to help teachers address variability in students' number knowledge.

For students to fully access the potential benefits of any NSK instruction or activity, it is vital for them to engage with mathematics (Baroody et al., 2016; Lein et al., 2016; Middleton et al., 2017; Webb et al., 2014). Mathematics engagement, a complex construct composed of multiple components, is defined by Middleton and colleagues (2017) as:

> The in-the-moment relationship between someone and her immediate environment, including the tasks, internal states, and others with whom she interacts. Engagement manifests itself in activity, including both observable behavior and mental activity involving attention, effort, cognition, and emotion (p. 667).

We are particularly interested in students' observable in-the-moment behavioral engagement during mathematics discussions around a specific mathematics NSK activity, primarily because this type of engagement is malleable by teacher actions (Middleton et al., 2017). In-the-moment behavioral engagement changes based on surrounding factors, and over time may lead to the development of mathematics interest as a trait (Middleton et al., 2017). Research examining observable in-the-moment indicators of engagement may yield information to guide teachers in effective strategies and activities useful for developing short and long term mathematics engagement. The NSK activity described in this study was designed to increase second-grade students' engagement in whole-class mathematical discussions about NSK concepts.

Literature Review

Inclusive Tier 1 Number and Operations Instruction

Researchers have made important progress on the learning and teaching of early number and operations (e.g., Baroody et al., 2009; Carpenter et al., 1999; Clements & Sarama, 2009; Kamii, 2000) as well as interventions that improve performance in this domain (e.g., Bryant et al., 2011; Chard et al., 2008; Fuchs et al., 2013; Jordan et al., 2012). The current intervention research base focuses on intensive interventions for first-grade students (e.g., Bryant et al., 2011; Clarke et al., 2014), one-on-one tutoring for low-performing first-grade students (e.g., Smith et al., 2013), or interventions for first- or second-grade students with learning disabilities (e.g., Dennis et al., 2016; Fuchs et al., 2005; Fuchs et al., 2013; Laurillard, 2016; Valenzuela et al., 2014). Many of these intervention studies are aimed at systematic and explicitly delivered Tier 2 (targeted small-group) mathematics interventions (Doabler et al., 2016) for students with or at risk for mathematics disabilities. There is a need to better understand how students needing one-on-one interventions can engage with whole-class activities designed to include them. Effective class activities are needed to provide all students with opportunities to develop number sense whether they have emerging difficulties, perform on grade-level, or need challenges to extend their number sense foundations.

Instructional practices that support students' mathematics learning in Tier 1 settings include rich tasks (Smith & Stein, 1998), tasks with low floors and high ceilings to promote access for all students (Franke et al., 2007), productive classroom discussions (O'Connor et al., 2015), and teacher facilitation of connections among students' strategies and key mathematics ideas (Stein et al., 2008). Developing skills to enact these instructional practices is challenging (Anthony et al., 2015; Kazemi et al., 2009). There is a need to understand how well teachers implement these practices, and how students who struggle in mathematics engage in these types of learning environments.

Engagement in the Mathematics Classroom

Researchers examine engagement in varied ways within mathematics (Baroody et al., 2016; Lein et al., 2016; Middleton et al., 2017; Patahuddin et al., 2018; Webb et al., 2014). Engagement is not something that a student simply demonstrates or does not demonstrate; instead, engagement is contextually bound, influenced by curricula activities, teacher practices, and student interactions. The degree to which any student is engaged depends in part on the opportunities to engage that are presented to them (Gresalfi et al., 2009). Analyzing students' specific behaviors and teachers' particular actions may aid researchers in "uncovering contradictions and constraints in the environment that shape individual experience" (Watt & Goos, 2017, pg. 137). Examining engagement of students with a history of low mathematics performance is particularly important, as these students are at higher risk for inattentiveness and problem behaviors (Wu et al., 2014) and may be excluded from mathematics discussions (Parks, 2019). It is therefore important to examine the connection between student engagement and achievement, as well as specific teacher practices to encourage student engagement and participation.

Mathematics Engagement and Mathematics Achievement

Higher rates of engagement, measured through direct observations, teacher rating scales, or student self-report, are predictive of higher mathematics achievement (Baroody et al., 2016; Hughes et al., 2008; Lein et al., 2016; Middleton et al., 2017; Webb et al., 2014). Webb and colleagues (2014) evaluated the relationship between student engagement in mathematics discussions, teacher actions, and student achievement by analyzing video-recorded class sessions of six teachers. The researchers collected data on the degree to which students participated in mathematics discussions, and the level of detail students provided in explanations. They found significant correlations between engagement variables and higher student achievement, and that providing more detailed explanations was significantly correlated with higher student achievement. Additionally, students who engaged with their peers' ideas at a high level (e.g., adding on details to other students' explanations) had higher achievement than students who engaged with others' ideas at a lower level. These findings indicate that engagement overall, as well as students' degree of engagement, influences achievement (Webb et al., 2014).

Baroody and colleagues (2016) examined the connection between student engagement and the mathematics achievement of fifth-graders using teacher and student-reported engagement ratings and direct observation of engagement. The researchers found that teacher-reported engagement ratings and direct observation of engagement were associated with higher mathematics achievement, based on students' scores on an end of year grade-level assessment. However, student-ratings of engagement were not associated with mathematics achievement.

Lein and colleagues (2016) analyzed the relationship between engagement and mathematics achievement in the context of an intervention designed to improve seventh-graders' proportional reasoning. The researchers collected direct observation data of student engagement using momentary time sampling on a set of target students who represented high, average, and low achieving students, recording whether students were actively or passively engaged. The researchers found that engagement was a significant predictor of mathematics problem-solving and accounted for a higher percentage of variance than students' prior mathematics achievement. Additionally, high, average, and lower achieving groups of students all demonstrated fairly high rates of engagement, but higher and average achieving students had higher rates of engagement than lower achieving students. The researchers recommend that future research investigate the influence of instructional features, as well as teacher-provided opportunities to respond (Lein et al., 2016).

Teacher Actions to Facilitate Engagement

Teacher actions play a key role in facilitating student engagement (Hunter, 2017; Mitchell et al., 2017; Patahuddin et al., 2018; Webb et al., 2014). Increased engagement is associated with teachers providing open-ended tasks and questions (Patahuddin et al., 2018) and asking students to elaborate on their thinking and other students' ideas (Webb et al., 2014). When teachers increase their focus on describing their thinking about particular problems explicitly, student engagement and competence in providing similar descriptions also increases (Hunter, 2017). Additionally, engagement decreases when teachers simplify tasks to make them less challenging (Patahuddin et al., 2018).

Teacher Talk Moves for Facilitating Mathematics Discussion Engagement

Classroom discussions can promote students' engagement with mathematical ideas (O'Connor et al., 2015), and teachers' talk moves for facilitating mathematical discussions are critical for supporting learning in these contexts (Hufferd-Ackles et al., 2004; Michaels & O'Connor, 2015; Tabach et al., 2020). Teachers' talk moves are phrases or questions intended to elicit students' ideas and engagement in mathematical discussions (O'Connor & Michaels, 2019). Research and practitioner literature names various types of talk moves, including *revoicing, repeating, reasoning, adding on,* and *waiting* (Chapin et al., 2009), which press students to explain, reason, and justify as well as actively listen to and evaluate peers' mathematical ideas (Michaels & O'Connor, 2015; O'Connor et al., 2015). While talk moves open up conversational spaces for students to participate in academically productive math talk, orchestrating effective discussions that are inclusive of all students' engagement is challenging and complex (Michaels & O'Connor, 2015; O'Connor & Michaels, 2019; O'Connor et al., 2017). Michaels and O'Connor (2015), in an in-depth study of two teachers, found that even when teachers use talk moves that open discussions around students' thinking, it is not sufficient to ensure a coherent discussion. O'Connor et al. (2017) found that within classrooms with a culture of active participation, both silent and vocal active participants made similar achievement gains, refuting the hypothesis that silent students will do worse on learning measures. Engle et al. (2014) examined students' authority in discussions and how classroom norms can be constructed to facilitate equitable access and influence to the intellectual authority in classroom discussion. These studies indicate the need for continuing research about students' engagement—such as active participation, authority, agency, and influence—in whole-classroom mathematics discussions. While there is a logical connection between teacher actions and student engagement in the context of whole-class mathematical discussions, researchers need to examine whether the number of opportunities to engage, and whether specific instructional practices (e.g., talk moves), correspond with student engagement rates (Lein et al., 2016).

Purpose and Rationale

This study is part of a larger project evaluating the influence of a nine-week instructional treatment on students' NSK learning, particularly for students with a history of low achievement (Shumway et al., 2020). Due to the important role engagement plays in student access to mathematics, a key component of our study was examining how students who had low pre-test scores engaged with the NSK instructional treatment as one way to evaluate its effectiveness and accessibility to all learners. Therefore, in this study, we compared a group of target students' rates of engagement in NSK whole-class discussions with their NSK outcomes. Due to the role that teachers play in creating engaging mathematics instruction, we also evaluated teacher actions to determine which actions were potentially correlated with student engagement.

Methods

Using a mixed-methods design (Creswell & Plano Clark 2011), we assessed rates and patterns of engagement of a sample of second-grade students during the implementation of a nine-week whole-class NSK instructional treatment to explore the relationship between similarities and differences in engagement and NSK outcomes. The research questions were:

1. What are the variations in students' active/passive engagement rates as they engage in a NSK instructional treatment?
2. What is the relationship between engagement rates and NSK outcomes over the course of a NSK instructional treatment?
3. What specific behaviors did students with the highest and lowest rates of active engagement display during the NSK instructional treatment that may have contributed to differences in NSK outcomes?
4. What teacher instructional practices were associated with student engagement rates during a NSK instructional treatment?

Participants and Setting

This study included five teachers and 75 students from five second-grade classrooms (all consented to participate) located in two public elementary schools and one charter school in the western U.S. The public elementary schools were located in a district that enrolled 6,002 students of whom 14.2% were identified as having disabilities, 11.5% were English Language Learners, and 22.3% came from households at or below the poverty level. Seventy-six percent of the students in the district identified as White, 15% as Hispanic/Latino, 4% as Asian, 2% as two or more races, and 1% each as Black, Hawaiian/other Pacific Islander, or other. There were 304 students enrolled in the charter school, 15.1% of whom were identified as having disabilities, 1.3% of whom were English Language Learners, and 30% of whom came from households at or below the poverty level. Eighty-one percent of the students in the charter school identified as White, 10% as Hispanic/Latino, 5% as two or more races, 2% as Hawaiian/other Pacific Islander, and 1% each as Black or Asian.

The students participated in regular mathematics instruction with the added component of the NSK instructional treatment at the beginning of their lesson. Of the 75 participants, six students were selected as case studies for in-depth analysis, based on their low pretest scores (one student per class in four classes; two students from a fifth class) on the NSK pretest. While this score does not define a student's overall mathematics abilities and achievement, we chose to focus on a narrow aspect of students' mathematics learning (i.e., NSK).

Of the six students, four are female and two are male. We did not collect demographic information on individual students. Throughout this paper, we refer to these six cases as *target students*. We collected additional data on these target students, including direct observations of their classroom engagement compared to peers in the same classroom (referred to as *comparison students)*.

Procedures

We obtained university Institutional Review Board approval prior to conducting research activities. During the pre-treatment phase, we administered NSK pretests to students in each class and provided professional development to participating teachers to prepare them to implement the instructional treatment. In the instructional treatment phase, teachers implemented the instructional treatment three days a week for nine weeks (27 sessions total), and we provided two follow-up teacher coaching sessions. We collected engagement data for the six target students once per week through in-person observations during instructional treatment sessions, and administered posttests in the post-treatment phase. We analyzed specific student engagement behaviors and teacher actions using video-recordings of instructional treatment sessions.

The Number System Knowledge Activity

The NSK activity for this study was based on Quick Images, a common elementary mathematics classroom activity that encourages students to recognize and combine visual representations of quantities and use numbers flexibly. Quick Images involve the teacher showing an image of a quantity (often an arrangement of dots, see Figure 1) to students for 2-4 seconds, which encourages students to subitize and spatially group amounts, instead of count by ones.

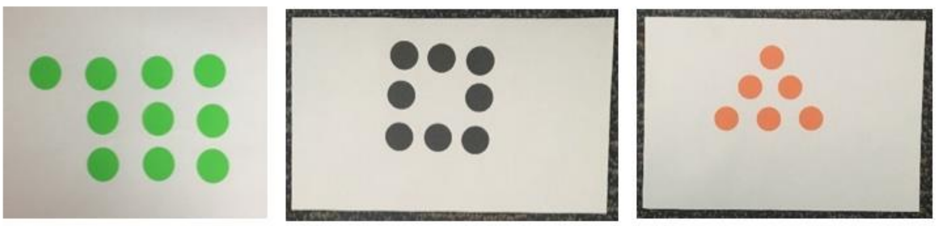

Figure 1. Example of Quick Images

In this study, students were then asked to describe the image and identify the total quantity they saw with partners and during whole-class discussions. The teacher guided the partner and whole-class mathematics discussions and emphasized connections between the quantities and symbolic representations (i.e., the NSK) by recording numerals and equations that represented students' verbalizations about the image and total quantity. During the NSK activities, the teacher asked open-ended questions to elicit a variety of student responses. For example, in response to "*how many and how did you know?,*" students expressed different ways to get to the same solution. A series of 27 sessions of Quick Images was purposefully sequenced to build on and gradually develop key NSK concepts. Each session included design features to promote access to all students, encourage multiple solution strategies, and provide opportunities for mathematical discussions.

Professional Development for the NSK Activity

The professional development (PD) for the five classroom teachers focused on learning about the 27 sessions of NSK activities and instructional practices central to enacting the activities (i.e., facilitating mathematical discussions and providing access to all students through discussions). The first PD session was prior to instruction. The researchers provided a binder of materials for implementing the activities and engaged teachers in rehearsal of the activities using mathematics talk moves (Chapin et al. 2009), which included co-watching videos of students/teachers engaging in similar activities, reading lesson plans for study activities, and role playing activities together. Finally, we discussed the key features and math talk moves embedded in each of the activities. The subsequent three PD sessions included individual coaching and group discussions about ways to use mathematics talk moves to facilitate engagement in purposeful discussions about flexible and interesting NSK strategies for solving the Quick Images. We encouraged teachers to see the Quick Images in a variety of ways (not just one correct solution), enabling them to ask prompting questions to support and build upon various student understandings.

Data Sources and Analysis

We used four main data sources for this analysis: 1) NSK Assessment (pretest and posttest), 2) engagement rates, 3) frequency counts of specific active engagement behaviors, and 4) frequency counts of specific teacher actions. The NSK Assessment is a reliable and valid NSK measure (Geary et al., 2009) composed of three subtests: number sets test (Geary et al. 2009), number line estimation tasks (Siegler et al., 2011), and computational fluency (Fuchs et al., 2003). Geary and colleagues found that these subtests defined a single NSK factor with α =.81, which better captures key variations in children's early mathematical development than mathematics achievement test performance.

Student engagement rate data consisted of a mixture of quantitative data (engagement percentages) and qualitative data (notes and video analysis of specific student behaviors during observations). Trained research assistants collected data on student engagement in-person using momentary time sampling during nine class sessions per teacher, recording the percentage of time each of the six target students was actively engaged, passively engaged, or off-task. Table 1 summarizes researcher-created indicators for active and passive engagement and off-task behaviors.

Using MotivAiders® (devices that vibrate at specific time intervals), we rotated in a systematic sequence every

20 seconds between observing target and comparison students, observing one student at a time. For each observation session, we summarized engagement data for each target student and for the comparison students overall as percentage of time actively engaged, passively engaged, and off-task. The lead author trained research assistants to collect engagement data by operationally defining the different categories of engagement (actively engaged, passively engaged, off-task) and providing specific examples of behaviors that fall within each category. The research assistants and lead author then practiced the data collection procedures by watching videos of NSK quick images classroom sessions until research assistants achieved at least 80% agreement with the lead author on observed behaviors.

We assessed Inter-Observer Agreement (IOA) for engagement data by having an independent observer watch 33.3% (15) of the 45 total video-recorded observation sessions, randomly selected and evenly distributed between the five teachers. The independent observer followed the same procedures using a MotivAider® device and momentary time sampling. A percentage of agreement was calculated using point by point agreement by dividing total agreements by the sum of total agreements and disagreements, then multiplying by 100. Average IOA was 71% (range 51-93%). We further summarized these data across all nine observations per student using descriptive statistics. To determine whether engagement was correlated with changes in mathematics achievement, we compared each student’s average engagement rates to the amount of change between their pretest and posttest.

Table 1. Indicators of Active Engagement, Passive Engagement, Off-Task Behavior

Active Engagement	Passive Engagement	Off-Task Behavior
• Raising Hand	• Watching teacher	• Off-topic talking
• Talking with a peer (on-topic)	• Looking at board	• Laying down
• Sharing with whole class	• Looking at peer (on-topic)	• Looking away from speaker
• Writing on individual/class dry-erase board (on-topic)	• Looking at/using materials (appropriately)	• Moving around (unprompted)

We video-recorded and analyzed one instructional treatment session per week per class. Using these video data, we recorded frequencies and types of active engagement behaviors. We calculated averages and ranges of frequencies for each student, summarized across all observations.

To determine potential connections between student engagement and teacher actions, we analyzed these same video-recordings to record how frequently each teacher used the talk moves (Chapin et al., 2009), and other actions to facilitate student engagement and participation. We used an iterative process of descriptive and process coding. First, we watched the videos and used the talk moves as an initial framework with which to code teacher behaviors. Two researchers independently coded each video, and met to discuss and resolve any disagreements in coding until achieving consensus. We coded other teacher engagement actions as they occurred, again meeting to discuss and resolve disagreements until achieving consensus. We recorded frequencies of instances that each teacher used the different talk moves, as well as types and frequencies of other actions designed to facilitate student engagement. The talk moves and teacher actions we observed, with definitions of each, are presented in Table 2.

Table 2. Teacher Talk Moves and Other Engagement Strategies

Teacher Actions	Definition	Example
Revoice	Restating something a student said to ensure understanding of their comment	"I heard you say that _____. Do I have that right?"
Repeat	Asking a student to restate something they or a peer just said	"Can you repeat what Lisa said, in your own words?"
Add On	Asking students to add on to others' comments	"Pablo said he noticed _____. Can anyone add on to that?"
Student Response	Calling on students to provide an answer or comment	"How many did you see and how did you see it?"
Pair Share	Having students talk in pairs to share their thinking	"Turn to a partner and tell them how many you saw and how you saw it."
Thumbs Up	Asking students to show a "thumbs-up" (or down) hand gesture to indicate agreement or disagreement	"Give me a thumbs-up if you saw two groups of five also."
Same or Different	Asking students to raise their hands if they had the same or different strategy shared by a peer	"Raise your hands if you saw this in the same way Lara saw it" "Raise your hands if you saw this in a different way than Lara saw it"
Whiteboard Use	Prompting students to write on individual or class dry erase boards, or a class Smartboard	"Write down the number you saw" "Come up to the board and circle how you grouped them"
Hand Signal	Any teacher-prompted gestures that were not thumbs up/down or raising hands	Hand movement back-and-forth to indicate agreement; snapping fingers in air to indicate a celebration; raising certain number of fingers to indicate different things

Note. Revoice, repeat, add on are talk moves (Chapin et al., 2009); student response, pair share, same or different, whiteboard use, and hand signal are other engagement strategies.

Results

Research Question 1: Variations in Rates of Active/Passive Engagement

To answer the first research question, we calculated the mean and range of active engagement, passive engagement, and off-task behavior for each target student, and target and comparison students collectively (Table 3). These data indicate on average, target students had similar engagement levels as comparison students. Target students had slightly lower active engagement rates and slightly higher passive engagement rates and off-task behavior versus comparison students. There were wide ranges of engagement within each category of engagement behavior, across all target and comparison students.

Table 3. Target Student Engagement

Student (Class)	Active (Mean)	Active (Range)	Passive (Mean)	Passive (Range)	Off-Task (Mean)	Off-Task (Range)
Pablo (2)	23%	12-44%	44%	23-60%	34%	16-55%
Lara (5)	31%	9-62%	58%	33-83%	11%	0-32%
Gillian (3)	19%	8-32%	65%	54-83%	15%	4-31%
Ada (4)	22%	9-36%	72%	45-92%	6%	0-18%
Nadia (1)	15%	7-36%	78%	57-91%	7%	0-35%
Trent (1)	5%	0-14%	45%	27-83%	49%	17-77%
All Target Students	19%	0-62%	60%	23-92%	21%	0-55%
All Comparison Students	25%	0-50%	58%	35-80%	17%	0-44%

Research Question 2: Relationship Between Engagement Rates and NSK Gains

To answer the second research question, we analyzed the target students' NSK assessment scores. Table 4 summarizes the NSK assessment scores for these six students, in order of greatest to smallest gains.

Table 4. NSK Pretest, Posttest, and Gain Scores

Student (Class)	Pretest (%)	Posttest (%)	Gain (percentage points)
Pablo (2)	22	51	29
Lara (5)	24	52	28
Gillian (3)	37	61	24
Ada (4)	31	54	23
Nadia (1)	32	49	17
Trent (1)	33	46	13
All Comparison and Target Students	42	63	21

The average gain among the 75 students was a 21 percentage-point test score increase. Four of the six target students made greater than a 21 percentage-point increase. The two students from Class 4 (Nadia and Trent) made less than a 21 percentage-point gain.

Next, we compared students' gains in NSK scores with their percentages of off-task, passive, and active engagement (see Table 5). Collectively, these data indicate that target and comparison students benefitted similarly from the NSK instructional treatment, and students with higher rates of active engagement made greater NSK gains between pre and posttest.

Table 5. Comparison of Rankings for Engagement and NSK Gains

Active	**Passive**	**Off-Task**	**NSK Gain**
Lara (5)	Nadia (1)	Trent (1)	Pablo (2)
Pablo (2)	Ada (4)	Pablo (2)	Lara (5)
Ada (4)	Gillian (3)	Gillian (3)	Gillian (3)
Gillian (3)	Lara (5)	Lara (5)	Ada (4)
Nadia (1)	Trent (1)	Nadia (1)	Nadia (1)
Trent (1)	Pablo (2)	Ada (4)	Trent (1)

Note. The class number for each student is indicated in parentheses.

Research Question 3: Specific Active Engagement Behaviors Related to NSK Gains

To answer the third research question, we analyzed video data to identify specific types of active engagement behaviors participants with the highest and lowest rates of active engagement demonstrated. Active engagement behaviors included students sharing with peers, interacting individually with teachers, and sharing with the whole class. We calculated percentages of each type of active engagement behavior out of the total instances of active engagement behaviors to compare across students. Table 6 displays the percentage of each type of active engagement behavior out of the total active engagement behaviors for each target student.

Table 6. Active Engagement Behaviors: Percentage of Each Type

Student (Class)	Total AE Behaviors	% AE Peers	% AE Teacher	% AE Class	% Time AE	NSK Gain
Lara (5)	24	50%	0%	50%	31%	28
Pablo (2)	25	36%	36%	28%	23%	29
Ada (4)	40	70%	2.5%	27.5%	22%	23
Gillian (3)	32	68.75%	0%	31.25%	19%	24
Nadia (1)	25	64%	20%	16%	15%	17
Trent (1)	15	53.3%	40%	6.7%	5%	13

Note: AE = Active Engagement; % AE Peers = percentage of AE instances interacting with peers; % AE Teacher = percentage of AE instances interacting individually with teacher; % AE Class = percentage of AE instances interacting with whole class; % Time AE = mean percentage of time spent actively engaged per session; NSK gain = point gain on the Number Systems Knowledge test.

First, with the exception of Pablo, the two students with the lowest percentage of active engagement behaviors and lowest NSK gains (Nadia and Trent) had the highest proportion of active engagement behaviors that consisted of one-on-one interactions with the teacher. Pablo was an outlier to this pattern; 36% of Pablo's active engagement behaviors consisted of one-on-one interactions with the teacher, and Pablo had the second-highest percentage of active-engagement behaviors. Based on analysis of the video recordings, Pablo's interactions with the teacher appeared to be distinctly different than the one-on-one teacher interactions of other target students. In particular, Pablo's teacher would go to Pablo first after giving students a prompt to share their thinking with a peer. In these interactions, it appeared that Pablo's teacher had Pablo share his thinking with her, and she would ask additional questions to help Pablo think through the task. The majority of Nadia and Trent's one-on-one interactions with their teacher (both from Class 1) were primarily related to behavioral prompts and reprimands, as opposed to engaging in an academic task.

Secondly, the student with the highest active engagement rate and highest NSK gain had the highest percentage of sharing with the full class, and the two students with the lowest active engagement rates and lowest NSK gains had the lowest percentages of sharing with the full class. While there were no clear trends related to the percentages of sharing with peers, the data indicate that students with a more even distribution of types of active engagement behaviors also had higher overall active engagement and NSK gains. The two students with the highest total frequency of active engagement behaviors (Ada and Gillian) were in the middle of the group of target students in regards to percentage of active engagement and NSK gains. However, Lara and Pablo, who had the highest percentage of active engagement and highest NSK gains, had lower total instances of active engagement behaviors.

Research Question 4: Association of Student Engagement Rates and Teacher Practices

To answer research question 4, we recorded the frequencies with which each teacher used the different talk moves and other engagement strategies, and then compared these data with students' engagement rates (Table 7). In addition to the talk moves, six major themes emerged in our coding of the videos across the teachers. These major themes consisted of other behaviors teachers demonstrated to facilitate student engagement, including student responses, pair shares, thumbs-up/down, same or different, alternative hand signals, and whiteboard usage.

Table 7. Teachers' Use of Talk Moves and Other Engagement Strategies

	Teacher 1	Teacher 2	Teacher 3	Teacher 4	Teacher 5
Teacher Actions	Total (M)	Total (M)	Total (M)	Total (M)	Total (M)
Revoice	81 (9)	73 (8.11)	61 (6.77)	52 (5.77)	60 (6.66)
Repeat	14 (1.55)	8 (0.88)	14 (1.55)	2 (0.22)	4 (0.44)
Add On	1 (0.11)	0	1 (0.11)	4 (0.44)	0
Student Response	134 (14.88)	106 (11.77)	93 (10.33)	98 (10.88)	171 (19)
Pair Share	19 (2.11)	22 (2.44)	22 (2.44)	29 (3.22)	11 (1.22)
Thumbs Up	27 (3)	28 (3.11)	20 (2.22)	25 (2.77)	9 (1)
Same or Different	12 (1.33)	10 (1.11)	13 (1.44)	13 (1.44)	4 (0.44)
Whiteboard Use	4 (0.44)	1 (0.11)	15 (1.66)	0	0
Hand Signal	3 (0.33)	15 (1.66)	3 (0.33)	9 (1)	2

Note. (M) = mean use per observed sessions; revoice, repeat, add on are talk moves; student response, pair share, same or different, whiteboard use, and hand signal are other engagement strategies.

Next, we compared the data on teacher actions to the student behavior and engagement data. Specifically, we were interested in comparing teacher actions intended to promote active student engagement and participation -- student responses and pair shares. Figures 2 and 3 present data on teacher actions compared with active student engagement behaviors of the students in the corresponding teachers' classes, reported as total frequency across all recorded class sessions.

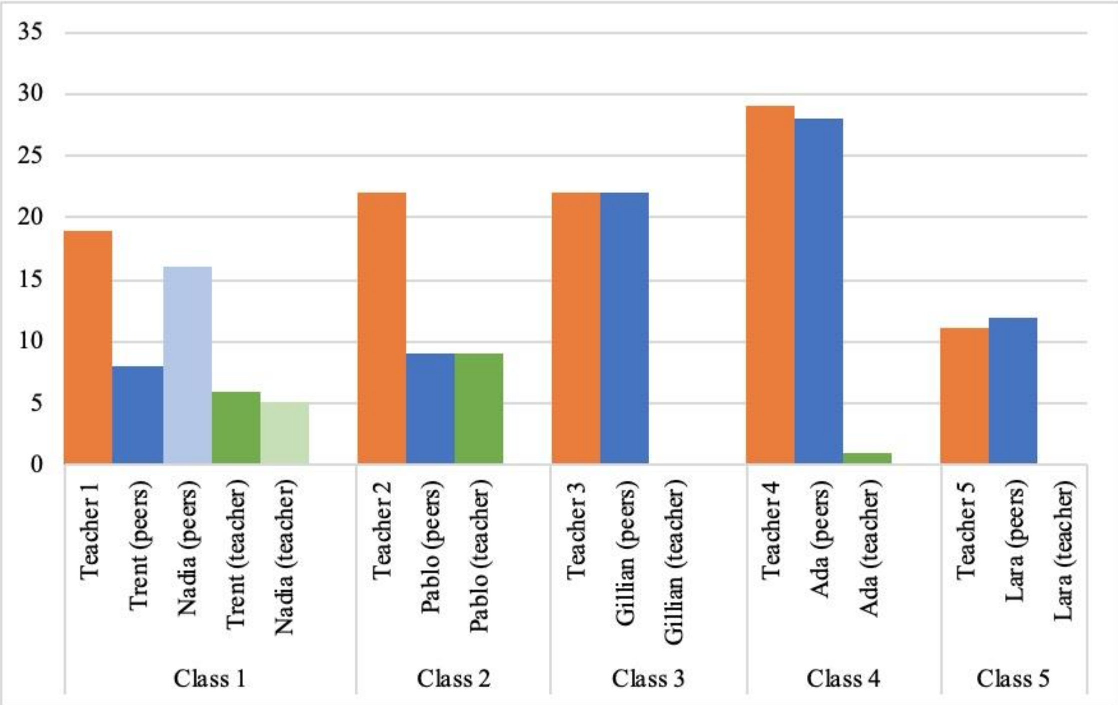

Figure 2. Frequency of Pair Shares with Teachers and Peers

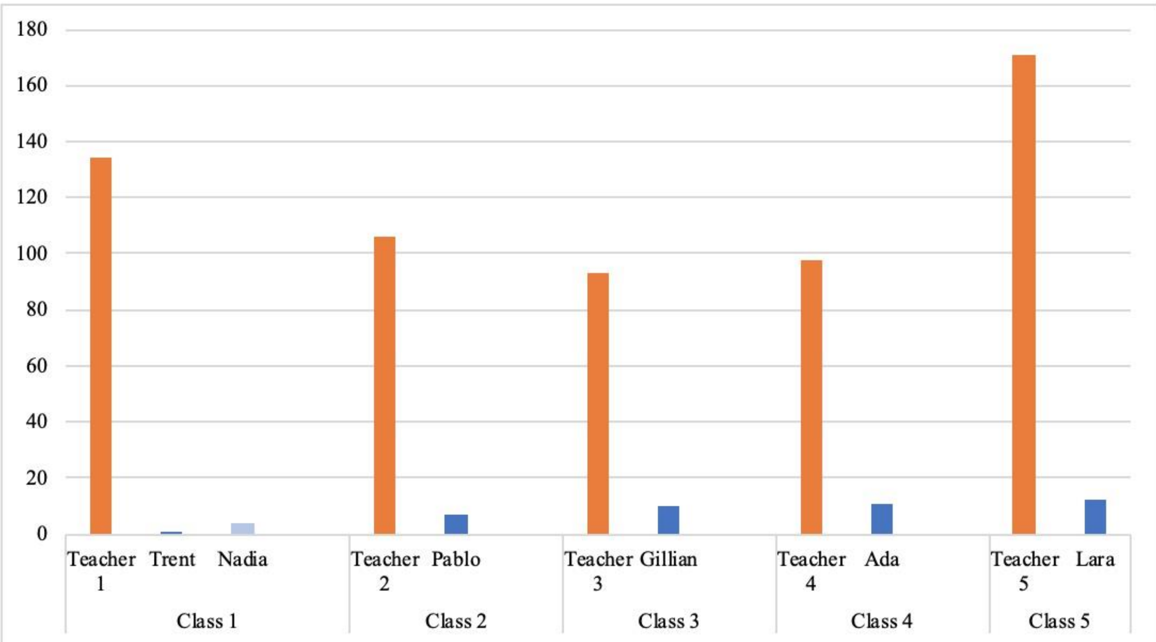

Figure 3. Frequency of Teacher-Elicited Responses and Target Students' Responses

The data on teachers' use of pair shares aligns with the data on students' sharing with peers. Teacher 4 had the highest frequency of pair shares, which aligns with Ada having the highest frequency of sharing with peers. Teachers 2 and 3 had the next highest frequency, also aligned with student data. Gillian (in Teacher 3's class) had the second-highest frequency of sharing with peers. Pablo (in Teacher 2's class) had the second-lowest frequency of sharing with peers, but also the highest frequency of sharing with the teacher. In the video-recordings, Teacher 2 would often go to Pablo immediately after giving students the prompt to share with a peer, and appeared to give Pablo opportunities to explain his thinking in scaffolded manner. Teacher 1 had the second-lowest frequency of pair shares, deviating slightly from student data; Nadia had the third-highest frequency of sharing with peers and sharing with the teacher. However, Trent (also in Teacher 1's class) had the lowest frequency of sharing with peers and second highest rate of sharing with the teacher. In the videos, Teacher 1's interactions with Trent appeared to primarily relate to behavior management, rather than giving Trent opportunities to share his thinking. Teacher 5 had the lowest frequency of pair shares, which deviates from student data. Lara (in Teacher 5's class) had the fourth-highest frequency of sharing with peers.

The data on teachers' use of student responses aligns with the active student engagement behavior of sharing with the whole class. Teacher 5 had the highest frequency of calling on individual students to provide responses to the full class. Lara, (in Teacher 5's class) had the highest frequency of sharing with the full class. Teacher 1 had the next highest frequency of student responses, deviating from the student engagement data on sharing with the full class. Nadia and Trent, both in Teacher 1's class, had the two lowest frequencies of sharing with the full class. Teacher 2 had the third highest frequency of student responses, and Pablo had the fourth highest frequency of sharing with the full class. Teacher 4 had the fourth highest frequency of student responses, deviating from student engagement data on sharing with the full class; Ada had the second highest frequency of sharing with the full class. Teacher 3 had the lowest frequency of providing opportunities for student responses, while Gillian had the third highest frequency of sharing with the full class.

In addition to examining how teacher actions aligned with students' active engagement behaviors, we compared the average rates of student engagement and improvements in NSK scores to identify teacher actions that may have contributed to higher engagement rates or academic improvement. The target students in classes 5, 2 and 4 had the highest average active engagement rates, and the target students in classes 5, 2, and 3 had the highest rates of improvement in NSK scores. Based on this, we evaluated specific differences in patterns of teacher actions for Teachers 2 and 5 compared with Teachers 1, 3, and 4. Teacher 5 had the highest frequencies of student responses and pair shares. Teacher 2 had the third-highest frequency of student responses, the second-highest frequency of pair shares, and the highest rates of thumbs up and alternative hand signals.

Discussion

The purpose of this study was to examine the engagement of students with low pre-test scores to evaluate whether they accessed a whole-class Quick Images number sense intervention at similar rates as their peers, and to evaluate teachers' implementation of the talk moves and other strategies to actively engage students. Overall, the data indicate that students with low pre-test scores had similar rates of engagement and NSK gains as their peers, and teachers were able to effectively implement several teaching practices to engage students during the instructional treatment. The results of this study contribute to the research base examining the relationship between direct observations of student engagement and mathematics achievement (Baroody et al., 2016; Lein et al., 2016). We examined students' engagement during a high-quality whole-class instructional practice designed to engage all learners, how student engagement aligned with NSK outcomes, and what teacher actions may have correlated with student engagement. While this study has limitations, the results warrant further examinations of the relationship between student engagement and mathematics achievement. This is an especially important area to examine with a focus on lower-achieving students in the context of whole-group instruction to ensure that tasks are accessible to and engaging for all learners. Three key findings emerge from the results.

First, students with higher rates of off-task behavior made lower NSK gains, and conversely, students with higher rates of active engagement made higher NSK gains. This finding supports prior research suggesting a connection between engagement and achievement in mathematics (Baroody et al., 2016; Hughes et al., 2008; Lein et al., 2016; Middleton et al., 2017; Webb et al., 2014). While prior research demonstrated that the degree to which students engage with one another's ideas influence mathematics achievement (Webb et al., 2014), the results of this study add to the research base by demonstrating that students' overall rate of in-the-moment engagement may impact achievement.

Second, teacher actions may be associated with active student engagement. The target students in classes 2 and 5 (Pablo and Lara) had the highest achievement gains and the highest active engagement rates. Teacher 5 had the highest frequency of student responses and pair-shares, and Lara (in class 5) had the highest active engagement rate. Teacher 2 had the highest rate of thumbs-up or other gesture responses, the second highest frequency of pair-shares, the third-highest frequency of student responses, and Pablo (in class 2) had the second highest overall active engagement rate. Students had higher rates of engagement in classes in which teachers used more strategies to facilitate engagement; these high engagement rates were associated with higher achievement gains, as evidenced primarily by Teacher 2's actions and Pablo's gains. This finding, combined with the association between engagement rates and NSK gains, demonstrates that it may be particularly important for teachers to implement strategies to facilitate active engagement and a variety of engagement strategies to boost student success. Notably, Teacher 2 had the highest rate of thumbs-up and gesture responses, which are efficient and easily implemented strategies that can be combined with more in-depth strategies (e.g., talk moves, having students describe their reasoning, and pair-shares) to enhance students' engagement. The findings of this study indicate a potential connection between specific teacher actions and student engagement, supporting previous calls for research examining such connections (Lein et al., 2016).

Finally, specific types of behaviors may have more of an impact than total percentage or frequency of active engagement on NSK gains. In particular, students sharing with the full class may be tied to higher gains, and individual interactions with the teacher may be tied to lower gains, depending on the characteristics of the teacher interaction. For example, Pablo (in Teacher 2's class) had the highest frequency of interacting with the teacher, but these interactions were academic in nature and Pablo had the highest gain in NSK scores. This is in contrast to the target students in Teacher 1's class (Nadia and Trent), who had the next highest frequency of teacher interactions. Nadia and Trent primarily had behavioral interactions with Teacher 1 (i.e., prompts, redirects, or reprimands), and had the lowest NSK gains. These data indicate that the particular one-on-one interactions students have with their teacher may influence engagement and achievement. This finding is consistent with prior research related to teachers' influence on student motivation, which indicates that teacher interactions high in emotional and instructional support tend to positively influence student motivation (Durksen et al., 2017). In this study, Pablo may have demonstrated higher rates of engagement and achievement in part because his teacher provided high levels of emotional and instructional support during their one-on-one interactions. The data in this study indicate that students who shared with the full class had higher active engagement rates and higher gains in achievement. This finding is consistent with prior research that found higher achievement and engagement associated with students explaining their thinking and comparing/contrasting their thinking with others (Hunter, 2017; Patahuddin et al., 2018; Webb et al., 2014). The findings of this study can be used to help guide teachers in ways to thoughtfully and strategically promote active engagement behaviors during mathematics discussions.

Limitations and Directions for Future Research

While the results of this study contribute to the research base on engagement in mathematics, they should be considered in light of several limitations. First, the study included a small sample size (six target students), and therefore more conclusive statistical analyses were not possible. However, having a small number of participants allowed us to analyze each target student's data quantitatively and qualitatively, and explore connections between engagement rates, specific behaviors, and teacher actions. Based on the findings of this study, we recommend that researchers explore similar questions related to how students engage in mathematics instructional activities, using a larger sample size and statistical analyses.

A second limitation of this study is that we only examined students' in-the-moment behavioral engagement through classroom observations. Classroom observations are an appropriate and useful way to examine students' in-the-moment behavioral engagement (Lein et al., 2016; Middleton et al., 2017), but they do not readily measure students' emotional or cognitive engagement, nor provide access to aspects of students' internal trait-based engagement (e.g., interests, motivations, or mathematics self-efficacy). However, examining students' in-the-moment behavioral engagement is valuable because classroom engagement is more immediately and easily influenced by teacher actions (Middleton et al., 2017). To better understand the complexity of engagement and variables that may influence it, it is important to examine multiple components of engagement (Middleton et al., 2017).

Researchers should aim to incorporate multiple measures of engagement in future studies. One particular framework that may aid researchers in doing so is the integrated framework of teacher-student interactions in mathematics developed by Durksen and colleagues (2017) to qualitatively evaluate "teacher-level supports that influence students' motivation and engagement in mathematics" (Durksen et al., 2017, pg. 176). Examining students' engagement and motivation during NSK activities using Durksen and colleagues' framework (2017) would yield more detailed data useful for guiding teachers in the implementation of effective strategies to engage all learners. Additionally, analyzing transcripts of classroom instructional sessions would provide valuable information related to students' overall engagement and how it may correspond with teacher actions.

Third, this study is limited in that IOA percentages are lower than ideal. Overall average agreement was 71% for this study, which is an acceptable level of agreement because some of the behaviors observers were asked to record were more subtle (especially for passive engagement behaviors) in the context of the observed activities. It is possible that average agreement was lower than ideal because the primary observers collected data in person while the IOA data collector did so by viewing video-recorded sessions. In the future, researchers should collect primary and IOA data in the same medium, if possible.

An additional limitation is that we were not able to collect teacher demographic data (e.g., number of years teaching experience, educational backgrounds, previous exposure to PD related to facilitating mathematics discussions or teaching NSK concepts). Teacher characteristics may have influenced students' engagement with the NSK activities, or their NSK achievement gains. In the future, researchers should examine whether teacher characteristics influence student engagement and achievement in mathematics.

Implications for Practice

While this study's limitations constrain the conclusions we can draw about the impact of student engagement on NSK achievement and the connection between teacher actions and student engagement rates, there are important implications for practice. Overall, we encourage mathematics education researchers and practitioners to purposefully incorporate instructional activities that promote *active* student engagement, especially when working with students with a history of lower mathematics achievement. Educators can do so by enacting practices such as the talk moves (Chapin et al., 2009). Talk moves that encourage students to explain their reasoning to the class, and engage with the reasoning of other students may be particularly powerful (Webb et al., 2014). Teachers should also implement low-effort strategies such as thumbs-up or other gesture responses to maximize engagement, especially combined with the talk moves.

One way to help students who face difficulties in mathematics may be to incorporate specific scaffolds into mathematics discussions, such as those that Pablo's teacher used. Teachers may promote the active engagement of students with a history of lower mathematics performance by serving as a partner during pair-shares, posing specific task-related questions and ideas when doing so. An additional scaffolding strategy that may accomplish a similar goal would be using strategic pairing and/or grouping during mathematics discussions, so that students with a history of lower mathematics performance or engagement are paired with students who have similar patterns of achievement and engagement (to more easily facilitate differentiated instruction), or with students who have different patterns of achievement and engagement (to build in peer supports).

The results of this study support the need for continued research in the complex area of measuring and designing activities to promote mathematics engagement. Our findings indicate that students with low pre-test scores had similar rates of engagement as comparison peers, active and varied mathematics engagement may influence student achievement, and specific teacher actions may facilitate active engagement more so than others.

Notes

This research was conducted with financial support from Utah State University's Office of Research and Graduate Studies under the Grant-Writing Experience Through Mentorship grant program.

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