The Impact of Written Feedback in Geometry Problem Solving through a Gallery Walk

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The Impact of Written Feedback in Geometry Problem Solving through a Gallery Walk

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Abstract

Traditionally, students experience difficulties solving problems involving geometric shapes and their properties. In line with the current curricular guidelines, it's important to reflect about the use of active learning strategies, which directly engage students in meaningful mathematical activity, that contribute to reverse this situation. This paper refers to a study that aims to understand the influence of peer written feedback on students' performance in a Gallery Walk context, in the scope of problem solving involving areas of plane figures. We followed a qualitative, interpretative approach and collected data in a 5th grade class, with 19 students, through participant observation, a questionnaire, interviews, written productions and photographic records. Results show that, when the sender of feedback is a student, different types of feedback are used, however not all of them promote the improvement of the receiver's performance. It was noticed that the most effective feedback is focused mainly on the emerging mathematical content, commenting on the student's performance in terms of task solution, process and results. It was also found that the feedback must be clear and simple so that the receiver clearly understands what the sender wants to convey.

Introduction

Geometry is an area of recognized relevance in Mathematics. It plays a fundamental role in children's development, not only in terms of the geometric perception of the environment that surrounds them (Freudenthal, 1973), but also in the development of fundamental skills associated for example with critical thinking, intuition, (inductive and deductive) reasoning, among others (Jones, 2002). However, Geometry does not always have the spotlight in Mathematics classes, especially in the most elementary levels, despite the many benefits it can provide students, even when presented in an intuitive and informal way, aspect that can trigger bad results (Sinclair & Bruce, 2015; Vale & Barbosa, 2014) or a poor conceptual knowledge. Reflecting about the central mathematical topic in this study, the measurement of the area of plane figures, some aspects are pointed out as problematic in the literature, such as: difficulties understanding measurement concepts; relating these concepts and mobilizing them to solve problems; misconceptions and prevalence of a procedural knowledge about the measurement of areas; memorization, without understanding, of formulas for the areas of plane figures (e.g. Chappell &

Thompson, 1999; Huang & Witz, 2013; Kamii & Kysh, 2006; Maher & Beattys, 1986; Zacharos, 2006).

It is thus up to the teacher to promote an effective teaching, selecting diversified tasks and resources, covering multiple geometric representations, seeking to establish connections, but above all valuing the role of visualization (NCTM, 2014). It is necessary to adapt teaching-learning methods in order to accompany the evolution and needs of society, seeking to create opportunities for the development of knowledge but also of a set of skills considered essential in the 21st century. A possibility lies on the use of active learning methodologies that allow, on the one hand, to counteract the sedentary habits typical of today's students and, on the other hand, to instill collaborative work practices, keeping students cognitively, physically and socially involved (e.g. Edwards, 2015; Vale & Barbosa, 2021). A Gallery Walk is conceived as a strategy that fits these requirements, also contributing to the development of mathematical communication, critical thinking, as well as the ability to solve problems, either individually or collaboratively. Students are encouraged to solve problems in small groups, presenting them in poster format so that other colleagues can comment on their work and, finally, discuss the solutions as well as the comments made, in each group and with the whole class (e.g. Vale & Barbosa, 2018, 2021). In this context, with the writing and analysis of comments, the formulation of written feedback by peers naturally emerges as a learning opportunity looking for self-regulation, which can help students to better understand the contents, as well as to identify their own mistakes.

Based on the discussed ideas, we considered pertinent to carry out a study with 5th grade students who were working on the topic areas of plane figures, seeking to understand the influence of peer written feedback on students' performance in a Gallery Walk context. Following this problem, three research questions were formulated: (1) How can we characterize the students' performance when solving area problems in the context of a Gallery Walk?; (2) Which are the characteristics of peer written feedback in a Gallery Walk?; (3) How does peer written feedback influence students' performance throughout a Gallery Walk dynamic?

Theoretical Framework

The Concept of Area and its Measurement: Teaching and Learning Issues

This study focuses on the geometry strand, in particular, on the topic of area measurement. The teaching unit addressed by the participants involved remembering the calculation of the areas of the rectangle and the square and the deduction of the areas of the parallelogram and the triangle. It is therefore important to reflect about some teaching and learning issues that allow us to better understand aspects of students' performance in this domain.

Usually, in mathematics classes, students are faced with routine tasks of low cognitive level, with few experiences that aim to contribute to the development of higher order skills (Stein & Smith, 1998; Vale & Barbosa, 2014). This situation also arises in the teaching of the topic areas of plane figures. And to reinforce the impact of these options in students' learning, the existing literature highlights that an inadequate approach in the study of area measurement with emphasis on memorizing mathematical formulas can contribute to many of the difficulties inherent to this topic (e.g. Huang & Witz, 2011; Huang & Witz, 2013; Gurganus, 2022). Some authors (e.g. Huang & Witz, 2013; Gurganus, 2022) point out that the poor understanding of formulas associated to the areas of plane figures may be due specifically to the transition from the use of a counting strategy, used in the first years, to the need of a conceptual understanding of the formula. Now, if a teacher only asks for the memorization of a set of formulas, without explaining/deducing their source and applicability, it is likely that a large part of the students, despite having memorized them, will not be able to apply them in problem solving contexts, especially with non-routine tasks. Then teachers must go beyond the area formulas as a product, in order to strengthen students' conceptual understanding since the early ages (Clements & Battista, 2001; Huang & Witz, 2011). In this sense, and taking into account that the measurement of the area of a figure implies the comparison between a quantity of a physical magnitude with a similar one, it is essential that, in an early stage, students carry out tasks that allow them, over time, to reach a standardization of measurements and measurement processes until they understand the associated concept of the formula (Clements & Battista, 2001). So, the approach to calculating the areas of the square, rectangle, parallelogram and triangle, should start from what students already know, seeking to establish connections based on the geometric relationships between the figures (NCTM, 2000; Sinclair & Bruce, 2015). The determination of formulas for the areas of figures should be developed from the analysis of figures decomposable into squares, progressing to the understanding of the general term.

The ability of students to solve problems involving the calculation of areas is developed based on mathematical experiences that allow them to explore formulas in specific geometric contexts and reflect on the manipulation of figures, in a dynamic perspective (Huang & Witz, 2011; Presmeg, 2014; Vale et al., 2018). On the other hand, "when students subdivide, combine and transform figures, they are investigating the relationships between them" (NCTM, 2000, p. 192). For example, by folding and cutting a parallelogram, students can understand that it easily "transforms" into a rectangle, with the same base and the same height, and that the way to calculate the area will be the same as calculating the area of a rectangle. In the same way, to calculate the area of a triangle, through the visualization and manipulation of a parallelogram or a rectangle, students understand that the area of a triangle is half the area of a parallelogram or rectangle. According to the NCTM (2000) the notion that different figures can have equal areas can help improve the development of general formulas that determine the area of a given shape. As mentioned by Clements and Battista (2001), it is up to the teacher to propose multiple experiences of comparing areas, encouraging students to use their own strategies in order to find a norm that allows them to deduce a certain conclusion.

In the field of Geometry, and in particular in the study of areas, visualization plays a central role in its understanding, as well as in solving problems involving this topic. In fact, solving problems involving areas of plane figures can be an arduous task if the student has not understood the meaning of the contents explored. As this is a content with a strong visual and representative nature, in which everything can be translated through drawing, the option for the use of visual representations can help the student develop problem-solving skills. A student who is able to develop a geometric eye, that is, the ability to see geometric properties separating themselves from a figure (Vale et al., 2018) is more skilled at visualizing geometric properties, which can facilitate the solution of the proposed problem. However, in order for students to employ a visual/dynamic strategy (e.g. Presmeg, 2014; Vale et al., 2018), it is essential that the teacher provides them experiences of this nature while solving problems. In this sense, it is up to the teacher to diversify the approaches to a given task since different individuals process the same information in different ways and, in this sense, many students show preference for analytical methods, while others tend to reason visually (Vale et al., 2018). In turn, the diversification of representations contributes to the flexibility of thinking, also providing the opportunity to think in different ways, allowing the development of creative visual solutions (Vale et al., 2018). The realization that not all students think in the same way when it comes to problem solving, showing preference for analytical methods – emphasizing the use of algebraic representations, numerical and verbal; visual methods – using visual representations, such as figures, diagrams and/or tables; or mixed methods – presenting solutions with analytical and visual characteristics; reinforces the need to explore different strategies, establishing connections between them to facilitate conceptual understanding (Borromeo-Ferri, 2012; Presmeg, 2014; Vale et al., 2018).

Other difficulties are also inherent to the process of learning how to calculate the areas of figures, in addition to those already mentioned in the beginning of this section, for example related to the correct application of the measurement units associated with the tasks' contexts. As examples, it is common to find the inadequate choice of units, the indication of one-dimensional units for area and two-dimensional units for length, or even the confusion between measurement and measurement unit (e.g. Huang & Witz, 2013; Lavrador, 2010).

Written Feedback as a Learning Mediator

The requirements of today's society imply the development of a multiplicity of knowledge as well as an autonomous lifelong learning, aspects that demand an exploratory teaching practice, which places the student at the center of the teaching and learning process. In this context, students must be actively involved in the self-regulation of their progress, in an interactive and collaborative way, monitoring their reasoning and the level of goal achievement. The self-regulation of learning can be understood as an intentional act that directly contributes to the progression and/or redirection of learning, allowing the control and regulation of the learning processes, students' thoughts, motivation, behavior and learning environment (Frison, 2016; Santos, 2020). Zimmerman and Schunk (2011) refer that self-regulation processes may be triggered by multiple practices, but mainly through the use of feedback, either through the action of the teacher or through the use of peer assessment.

Several definitions of feedback have emerged in the literature, for example, for Hattie and Timperley (2007) the main goal of feedback is to reduce eminent discrepancies between learning objectives and task performance. Feedback must necessarily lead to any type of action, or a set of actions, that the student develops in order to improve his/her learning. Santos (2020) reinforces that feedback is an intentional dialogue that aims to help students overcome their difficulties, bringing them closer to the expected solution, and should therefore be descriptive, emerging only after the student has had the opportunity to reflect on a given task. This practice should be oriented to the process as well as to the product, highlighting the strengths and weaknesses in the (mathematical) activity, through comments focused on the processes applied or necessary to solve a task, as well as how to improve the presented solution (Barbosa & Vale, 2021; Katsberg et al., 2020). In this sense, feedback can be considered as a practice that intends to promote students' learning, through the elaboration of oral or written comments, that guide their productions and interventions towards a better achievement.

Feedback can assume different forms however the focus of this study is on written feedback. Despite the oral strand being the most used in the classroom, it is important to highlight some of the advantages of written feedback practices. As opposed to oral feedback, it presents a more individualistic, tangible and private perspective that may have a greater impact on students' learning (Barbosa & Vale, 2021) and can be revisited as often as necessary for a better understanding of what is being studied (Kastberg et al., 2020). For these reasons and for the fact that students have few opportunities to either receive and elaborate feedback in writing, we decided to emphasize the written component in this study.

Feedback can assume different characteristics and focus on different aspects of students' work. Hattey and Timperly (2007) distinguish four levels of focus regarding feedback: on the task; on the process; on self-regulation; and on the student. At the first level, feedback should focus on how the student solved the proposed task, reflecting whether it was well understood or solved and whether the answers are in line with what was intended. At the second level, feedback is focused on the strategies used, as well as on the learning processes involved in understanding the task. In turn, at the third level, feedback is concerned with the ability of students to self-regulate and self-assess their work, taking into account the required intentions, which may lead to adjustments. Finally, at the fourth level, feedback can be personal, being directed to the self and often not related to the performance in the task, which is based on a more affective dimension, usually translating into positive comments about the student, frequently associated with praise. The first three levels are the ones that seem to have the greatest impact on learning, being quite promising for the development and understanding of the proposed task. Santos and Semana (2015) state that feedback based on self-regulation encourages students to be more committed to a task, to reflect about their work, as well as to act in order to improve it. In turn, according to the same authors, student-directed feedback appears to be the least effective in the learning process, as it focuses more on personal assessments of the student as a person, not being related to task performance. Other authors also reinforce the existence of these levels of focus. Pereira (2008) distinguishes three types: (i) feedback about the result, which aims to highlight the characteristics of the production, analyzing only the result instead of the process; (ii) feedback about the process, which focuses on the quality of the strategies used; (iii) and corrective feedback, which provides judgments on the result, also providing notes that lead to reflection about the work carried out, allowing the improvement of future solutions. Nelson and Schunn (2009) sum up these ideas dividing written feedback into: affective feedback and cognitive feedback. Affective feedback is the one that conveys an emotional language as a compliment or a personal judgment, and can assume three different roles: (i) positive, when it defines an approval of the work and weaves praise; (ii) negative, when it presents an unproductive criticism; (iii) or neutral, when it characterizes a certain action. In turn, cognitive feedback has six key characteristics: (i) summary of the main points of the work; (ii) identification of what needs to be improved; (iii) improvement suggestions; (iv) explanation of ways to solve the task in question; (v) posing questions that make it possible to clarify the tasks' solution; (vi) and reflection about the work carried out.

According to Brookhart (2008) it is imperative to pay attention to three key aspects in the elaboration of written feedback: Clarity, Specificity and Tone. Clarity alludes to maximizing the understanding that students must make of the information transmitted in the feedback, being essential that the sender uses clear and simple vocabulary, according to the learning level of each student, thus guaranteeing the understanding of the comments by the receiver. As reported by Terroso et al. (2019) the teacher must adapt the language, vocabulary and content of what he/she communicates to each and every student for whom the feedback is intended. As for Specificity, effective feedback should clearly convey the aspects that students need to pay more attention to, and it cannot be very extensive. Finally, Tone refers to the way in which the feedback is heard by students, which can stimulate or discourage their self-regulation.

The syntactic structure of written feedback is another aspect to consider, as it can be presented through questions, statements and/or symbols (Bruno, 2006). An interrogative feedback tends to be the most effective in the learning process because it induces the student to reflect on a given idea. However, if you just answer the questions asked without invoking reasoning, this form of feedback will not be effective. The same can happen when using statements and/or symbols, since the student can limit himself to reading the comments without resorting to reflection seeking for improvement. Santos and Semana (2015) also mention the use of feedback with a mixed syntactic structure, that is, with simultaneous predominance of interrogative and affirmative feedback. This can occur when the sender intends to identify what needs to be improved and then provide clues, in the form of questions, that promote the refinement of the work carried out. However, as pointed out by Bruno (2006), for a better understanding of this practice, it is essential to use an accessible, concrete language, according to the specificities of the educational context and that is directly related to what is happening.

However, balancing the amount of information emitted in the feedback is also an aspect to consider in its effectiveness. If it is too short, it may not be enough for the student to be able to understand what can be improved, on the other hand, if it is too long, the learning possibilities may also be reduced, as providing solutions that are too complete will not promote students' critical thinking and, consequently, they will not regulate their learning. According to Santos and Semana (2015), we should balance the information to be given, taking into account that feedback should not provide the answer, but only contain the necessary information for the student to be able to progress.

We can say that written feedback is an essential element in the process of formative assessment, as it helps with self-regulation of teaching and learning. That being said, based on the previously discussed characteristics, effective feedback should focus on the product, on the process and on the self-regulation of learning, through the formulation of judgments and comments that encourage students to reflect, so they can complete and/or improve their solutions. In addition to these aspects, feedback, which should be neither too short nor too extensive, should point out clues for future action, but also highlight positive aspects of the work developed to encourage students' engagement. Written feedback should evidence respect for the students and their work, position them as active and responsible agents of learning, leading them to think or question themselves, and should also be clear, point out clues to proceed and reanalyze the productions, informing what is already well done, not including error correction (Santos, 2020).

The Gallery Walk as a Facilitating Strategy for Feedback

To attain an effective mathematics teaching and learning, students must be actively involved in meaningful learning, having opportunities to communicate, reason, be creative, think critically, solve problems, make decisions and make sense of mathematical ideas (e.g. NCTM, 2014; Vale & Barbosa, 2021). The teacher can resort to a multiplicity of strategies that meet the requirements of active learning, triggering not only cognitive engagement, but also the social and physical dimensions. The Gallery Walk (GW) is an example of such strategies, in addition to being a vehicle for using feedback, either written or oral. This strategy allows students to collaboratively solve proposed tasks, presenting the solutions through a poster, fixed around the classroom, in a similar perspective used by artists when they exhibit their work in an art gallery (Vale & Barbosa, 2021, adapted from Fosnot & Jacob, 2010). This dynamic allows students to leave their chairs and move freely around the classroom to observe the work developed by their colleagues, as well as having the opportunity to share feedback and discuss ideas.

According to Vale and Barbosa (e.g. 2021) the work carried out along a GW can be divided into six fundamental steps (Figure 1): (1) Solving Tasks – students first solve the proposed tasks individually and, later, in group; (2) Construction of posters – they select the solutions to be presented and the way to organize them in a poster; (3) Presentation and observation of posters – posters are placed on the walls of the classroom, or outside, so that the other groups can analyze them; (4) Analysis and elaboration of comments – students write comments, doubts, questions,... on post-its and fix them to each poster; (5) Group discussion – each group collects its poster and analyzes the assigned feedbacks; (6) Collective discussion – the working groups present their solutions orally, respond to previously prepared comments, and a final synthesis is also carried out at this moment, reflecting on the fundamental knowledge that emerged.

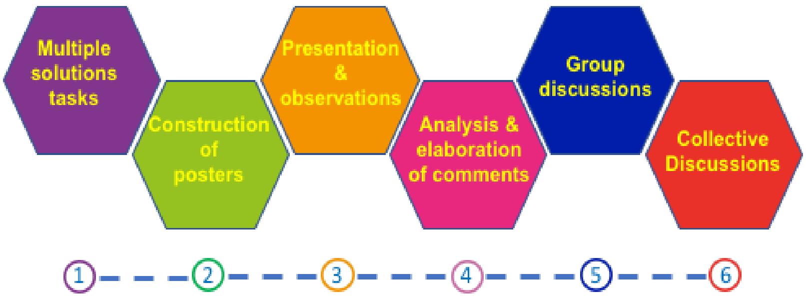

Figure 1. Steps of a GW (Vale & Barbosa, 2021)

The use of the GW promotes mathematical discourse, reflected in the way students represent, think, speak, question, agree/disagree in solving different tasks, as well as an opportunity for students to receive feedback about their work without fear of reprisals (Vale & Barbosa, 2021). Thus, it is possible to foresee a close link between communication (oral and written) and the opportunity for feedback throughout the steps that constitute a GW. Oral communication plays a very relevant role and can occur in multiple directions: from the teacher to the student(s), from the student to the teacher and from the student to the student(s) (Vale & Barbosa, 2018). However, despite this way of communicating being present in all phases of this learning methodology, written communication begins with the construction of posters and gains prominence soon after its production by the groups. The presentation of the tasks' solutions in the posters and the subsequent elaboration of comments are also ways of communicating, albeit through writing. Thus, it is up to the students to correctly explain what they intend to convey, so it is not misleading to the other students. Written feedback can be observed in a GW, through the formulation of written comments on the work developed in the posters. This feedback is central to the learning process, as it encourages reflection on previously solved tasks, allowing students to improve them. Oral feedback is also observable in the different phases of the GW, especially when it is necessary to comment on the work to be developed. This usually occurs from the teacher to the class, being more general than the feedback presented in the students' written comments.

To conclude, as an active learning strategy, a GW, promotes social interaction, as well as peer assessment, based on oral and written communication, inter and intra groups. For this to happen, a set of problematic situations are provided that allow students to understand, expand and deepen their mathematical knowledge. Through the use of this strategy, students assume a prominent role in the assessment process, allowing them to evaluate their peers, issuing written feedback on the work developed.

Methodology, Context and Participants

This study aimed to understand the influence of written feedback on 5th grade students' performance in the context of a GW and from a peer review point of view. Considering the problem defined, we chose to use a qualitative and interpretative approach (Erickson, 1986), grounding this decision in the fact that the main goal was to analize the perspective and reactions of the participants to a particular situation.

The research was conducted with nineteen 5th grade students, seven female and twelve male, aged between 9 and 11 years old. For the development of the fundamental parts of the study, despite the active engagement of the whole class, the students were divided into six groups to carry out the GW. One of the authors also assumed the role of teacher of these students, articulating the teaching practice with the data collection. The presence of the researcher in the context facilitated a more profound knowledge of the students and a more descriptive and holistic account of the facts evidenced during the classes. The students were organized into six working groups, five with three students and one group with four students. It should be noted that, to decide the constitution of the groups, we tried to join the students with more difficulties with others who presented a better performance in Mathematics. On the other hand, behavioral issues were also considered, strategically placing certain disruptive elements in specific groups.

At the time of the study, these students were working on Geometry concepts, namely areas and area measurement, focusing on the areas of the rectangle, square, parallelogram and triangle. The classes were organized around the five practices for orchestrating productive discussions, proposed by Smith and Stein (2011): Anticipating potential students' responses and solutions; Monitoring students' responses and solutions interactively; Selecting representative solutions for subsequent presentation; Sequencing students' solutions; and Connecting the students' strategies with the formal processes underlying the tasks. During the whole group discussions, students were able to contact with strategies of different nature, namely analytical and visual ones, to encompass the potential diversity of learning styles and also contribute to the expansion of their repertoire of strategies.

Data was collected in a holistic, descriptive and interpretative manner, during the classes, and included: participant observation, taking notes of the students' reactions and interactions; documents, namely individual records, the group poster with the solutions to the tasks proposed and the post-its with the feedbacks; a semi-structured interview to each group after the implementation of the GW to access the students' opinions about the GW dynamics and clarify details of the solutions; and photos. To analyze the data, we used a qualitative, inductive approach, recurring to content analysis (Miles & Huberman, 1994), relying on the written productions, the transcriptions of the interviews, the observations made, and the field notes. After repeatedly reading and consulting the collected information, we proceed to categorizations to systematize the data and facilitate the interpretation of the results. In this process, we formed the categories with the support of the problem and research questions, as well as the theoretical framework, which led to the following categories: Performance; Features of the written feedback. The categories were refined through the formulation of subcategories and the respective indicators, as shown in Table 1:

Table 1. Categories of Analysis

Categories	Subcategories	Indicators	References
Performance	Difficulties	Felt in the solution process; Evidenced in understanding the task statement; Applying the formula to calculate the area of the figures; Relating the concept of area with the specific properties of each geometric figure; Using proper measurement units	Clements & Battista (2001); Huang & Witz (2011, 2013); NCTM (2014)
	Strategies	Analytical Visual Mixed	Borromeo-Ferri (2012); Presmeg (2014); Vale et al. (2018)
	Correction of the solution	Correct Partially correct Incorrect Absence of solution	
Features of the written feedback	Focus	Student Task Process Result Self-regulation	Hattie & Timperley (2007); Nelson & Schunn (2009); Pereira (2008); Santos & Semana (2015)
	Domain	Clarity and correctness of the language; Mathematical content; Organization and presentation of the poster	Brookhart (2008); Terroso et al. (2019)
	Syntactic structure	Interrogative Symbolic Affirmative Mixed	Bruno (2006); Santos & Semana (2015)
	Extension	Short Long	Santos & Semana (2015)

The first category, related to students' performance, aims to reflect on the way they solved the GW tasks. At first, the students' difficulties were analyzed during the solution of the tasks, taking into account the ideas expressed in the literature (Clements & Battista, 2001; Huang & Witz, 2011, 2013; NCTM, 2000), reflecting on the difficulties: felt in the solution process; evidenced in understanding the task statement; in applying the formula that allows to calculate the area of figures; in relating the concept of area with the specific properties of each geometric figure; in using proper measurement units. Also in this category, we also analyzed the nature of the solution strategy which, according to Presmeg (2014), Borromeo-Ferri (2012) and Vale et al. (2018), can assume an analytical, visual or mixed form. Finally, the correction of the solution is also a focus, recognizing if it is correct, partially correct, incorrect or if there is an absence of a solution. It should be noted that, although there are no literary references that support this subcategory, a solution will be considered correct whenever a group presents a clear and mathematically correct reasoning, even when the students do not contextualize the calculations presented or don't represent the measurement units, exhibiting only the answer to the problem.

The second category, which has its subcategories grounded in the literature, refers to the features of the feedback formulated by the students during the GW, in terms of focus, domain, syntactic structure and extension. In this sense, in the data analysis, and according to the literature (Hattie & Timperley, 2007; Nelson & Schunn, 2009; Santos & Semana, 2015), regarding the focus, feedback can be applied to: the student's performance in the orchestration of the work developed, normally occurring with the use of praise; the task, alerting the recipients to the understanding and the correction of the task, and should also highlight if they present a response according to what is intended; the strategies used and the processes involved; the result, examining the product of the presented task. In turn, the feedback provided can also focus on the domain of clarity and correctness of the language, the mathematical content (Brookhart, 2008; Terroso et al., 2019) and even the organization and presentation of the poster, in the context of a GW. Written feedback can present different syntactic structures: interrogative, when students are questioned about a specific aspect; symbolic when feedback is presented through drawings and/or symbols; affirmative, using sentences; and even in a mixed form (Bruno, 2006; Santos & Semana, 2015). Finally, we also included the extension of feedback, referring to whether it is short or long (Santos & Semana, 2015).

Results and Discussion

The GW was implemented using the model proposed by Vale and Barbosa (2018, 2021), following its six steps, and had the duration of 2h30m. The first step, task solution, began with the presentation of two tasks that the students had to solve, first individually and, later, compare and discuss the strategies within the group, selecting the one(s) they intended to present in the poster. The tasks aimed to address the areas of the rectangle, square, parallelogram and triangle. The selection of the tasks was guided not only by the need to diversify the content, but also by the level of cognitive demand and its' adequacy to the work developed with the students at the time, allowing multiple solutions, namely of analytical but also visual nature.

The tasks proposed in the GW, which are presented in Figures 2 and 3, comply with the abovementioned conditions, the first with a lower level of cognitive demand and the second with a higher level of cognitive demand.

In the figure you see a representation of the land owned by Mr. José.
Calculate the area of the land in m^2.

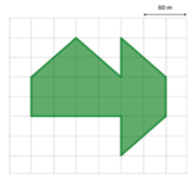

Figure 2. Task 1

Rectangle [ABCD] was constructed by joining, through one of the sides, two geometrically equal squares. Each square has a side with a length of 5 cm.
In this rectangle, a blue parallelogram was drawn in which each vertex is the midpoint of one of the sides of the initial squares.
Calculate the measurement of the area of the parallelogram, in cm^2.

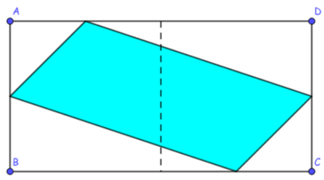

Figure 3. Task 2

In the following step, poster construction, the groups were instructed to reproduce their solutions in the posters, organizing the content as clearly as possible. Then, in the presentation and observation of the posters, the different groups exhibited their work in the corridor next to the classroom and the students, individually, walked through the gallery in order to observe the solutions proposed by their colleagues. The fourth step of the GW corresponded to the elaboration of comments, in which the students were asked to observe each poster and weave their personal comments on post-its. We made available post-its of different colors to differentiate the comments referring to the aspects to be improved from those that highlighted positive aspects of the work presented. After this step, and all the students having written the comments to the different posters, they were collected so that each group could analyze the content of the feedback provided by the colleagues to improve the work developed, if they wished to do so. Finally, in the collective discussion, with all the posters displayed on the classroom board, each group was called by the teacher to explain the solutions presented, justify their choice and react to the comments written by the colleagues. During the discussion, all students could intervene, taking advantage of the moment to clarify any doubts that arose when observing the posters. Still at this stage, the teacher took the opportunity to highlight some aspects related to the mathematical content, mainly in terms of the clarification of the reasoning, making a final synthesis of the main concepts. Figure 4 illustrates the implementation of the GW's steps.

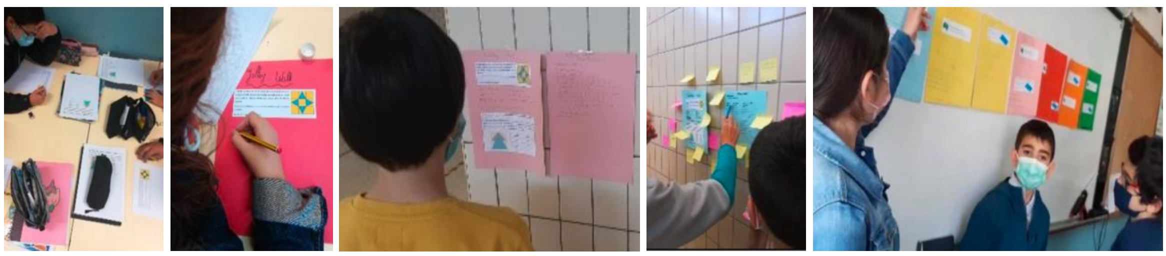

Figure 4. Implementation of the GW

To present and discuss the results, we chose to follow the steps of a GW and, throughout them, analyze the students' performance and the characteristics of the written feedback, taking into consideration the two main categories emerging from the research questions.

Steps 1 and 2: Solving Tasks and Construction of Posters

The class, organized in groups, initially contacted with both tasks. The students started by reading the statements, and, when they felt difficulties regarding the conditions of a problem, they either solved them within the group or looked for the teacher's guidance. After understanding what was asked in each problem, the students discussed possible strategies to get to the solution:

> BA: We have to divide the green part! [Group 1; Task 1]
> MC: Oh? How? [Group 1; Task 1]
> BA: Using figures we already know! [Group 1; Task 1]

> EM: [Pointing to the figure] If the side of the square measures 5 cm, then the side of the rectangle measures 10. The parallelogram should have the same area as the rectangle. [Group 2; Task 2]
> LA: That can't be, because the base and height are not the same! [Group 2; Task 2]
> AC: [After a while] This is easier than it looks! The figure is not only the parallelogram, it also has triangles. [Group 2; Task 2]
> LA: I was thinking of calculating the area of the rectangle and then subtracting the area of the triangles! [Group 2; Task 2]

To facilitate reasoning, students chose to solve the tasks individually, and only after this moment decided among themselves which strategies would be correct and most suitable to present in the poster. Following this step, the groups proceeded to the construction of the poster, making decisions about its organization and also about the content. Analyzing the solutions selected by each group to present in the poster, we noticed the option for different strategies, both in tasks 1 and 2. Table 2 summarizes the nature of the strategies used by the different groups to solve the two tasks proposed. In the first task, only groups 1 and 3 chose analytical approaches, while the remaining groups preferred to use visual or mixed solutions. However, group 2 presented two solution strategies, one of visual nature and another of analytical nature, which they decided to include in the poster, showing two different ways of approaching the same problem. Task 2 did not generate as much diversity of strategies, possibly because it presented a higher level of cognitive demand than the previous one and these students felt more comfortable with analytical methods, due to their prior experiences. Only one group chose a visual solution.

Table 2. Summary of the Strategies used

Tasks	**Strategies**	**Groups**					
		1	**2**	**3**	**4**	**5**	**6**
1	Analytical	x	x	x			
	Visual		x			x	
	Mixed				x		x
2	Analytical	x	x	x	x	x	
	Visual						x
	Mixed						

In the first task, the groups that used analytical strategies divided the figure in triangles and rectangles, calculating the areas of each figure through the use of the respective formula, then they calculated the total area. The solution presented by group 3 in Figure 5 represents an example of this approach:

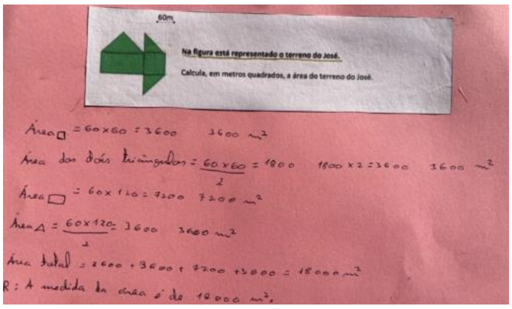

Figure 5. Solution of Task 1 presented by Group 3

The visual strategies used by groups 2 and 5 and the mixed strategies used by groups 4 and 6 were similar. In the case of the visual approach, the students first defined as unit area a 2x2 square (measure of four-unit squares), corresponding to 3600 m^2. Then they analyzed how many squares of this type there were in the figure, verifying that there were 5. In the case of the mixed strategy, there was a predominance of the visual approach, recurring to a dynamic solution. The students reorganized the “parts” of the figure to transform it into familiar figures, two squares, facilitating the calculation of the total area. This way, they started by joining the triangles, obtaining a square with a side length of 120 m and a square with a side length of 60 m. Figure 6 presents an example of a visual strategy (group 2) and another of a mixed strategy (group 4).

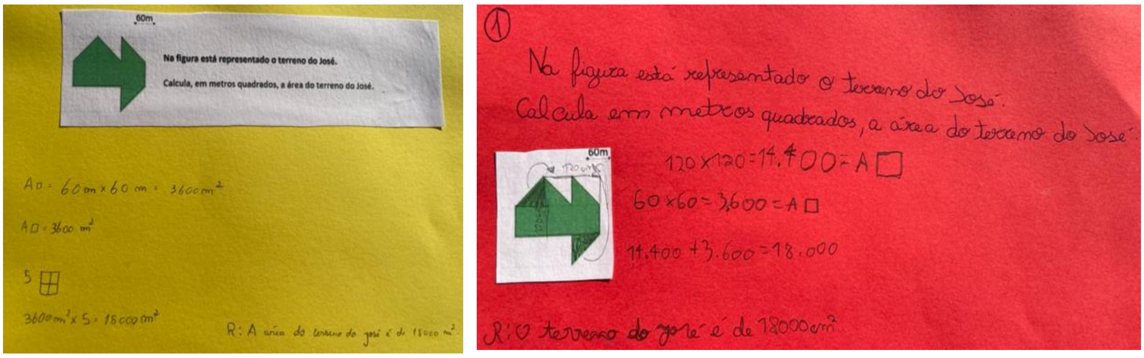

Figure 6. Solution of Task 1 presented by Groups 2 (left) and 4 (right)

As for the second task, groups 1, 2 and 5 used an analytical strategy. They started by determining the area of the white triangles, realizing that there were two pairs of congruent triangles. Then they calculated the area of the rectangle so that they could subtract the areas of the triangles, thus obtaining the area of the intended parallelogram. Figure 7 includes as an example the solution presented by group 2. Group 6 was the only one to use a visual solution. The students realized that by moving parts of the parallelogram (one of the triangles obtained by decomposing the figure by its diagonal), they would be able to fill half of the rectangle, as shown in Figure 8. After discovering that the area of the rectangle is 50 cm^2, they inferred that the area of the parallelogram would be 50÷2=25 cm^2.

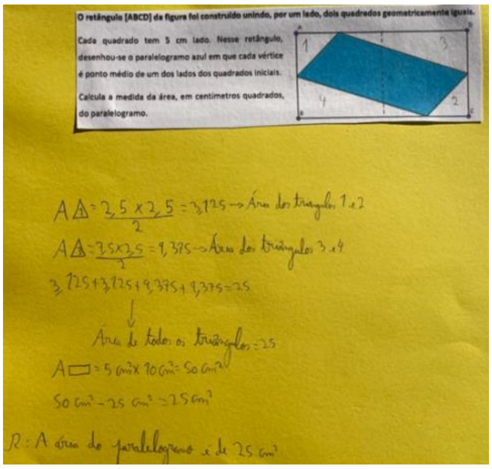

Figure 7. Solution of Task 1 presented by Group 2

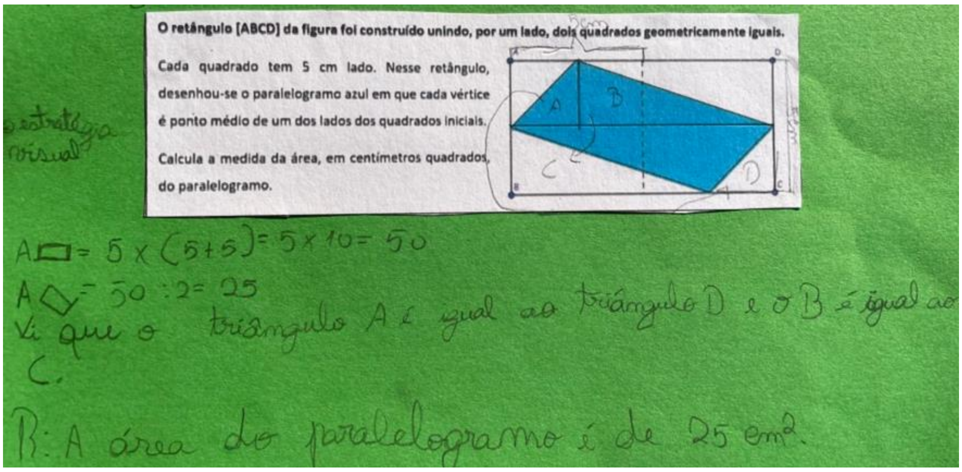

Figure 8. Solution of Task 1 Presented by Group 6

Although the students showed greater flexibility of reasoning in the use of visual and/or mixed solution strategies, recognizing the generation of simpler solutions, they showed a clear preference for the application of analytical strategies, especially in the second task, of a higher level of cognitive demand. This situation can be explained by the low visibility given to visualization and visual approaches in the academic path of these students.

As shown in Table 3, all the groups attempted to solve the tasks and presented at least one solution. Task 1, being of a lower level of cognitive demand, resulted in mostly correct solutions with only two partially correct solutions being presented by groups 1 and 6, due to the difficulty in correctly expressing the applied mathematical knowledge, namely not all calculations were contextualized or clearly associated with the figures resulting from the decomposition. On the other hand, in task 2, we found three different scenarios: groups 2 and 6 with correct solutions; groups 3 and 4 with incorrect solutions, showing difficulties in understanding the statement and conditions of the task, which led to incorrect assumptions about the properties of the parallelogram as having the same dimensions as the rectangle or choosing to decompose the parallelogram in squares and triangles with incorrect dimensions; and group 1 with a partially correct solution due to the inadequate writing of some equalities, revealing lack of attention with the scientific correctness of written communication.

Table 3. Summary of the Correction of the Tasks' Solutions

Tasks	**Correction**	**Groups**					
		1	**2**	**3**	**4**	**5**	**6**
1	Absence of solution						
	Incorrect						
	Partially correct	x					x
	Correct		x	x	x	x	
2	Absence of solution						
	Incorrect			x	x		
	Partially correct	x				x	
	Correct		x				x

During the first two steps of the GW, some difficulties were noticed, particularly while solving the tasks. A common situation arose in terms of understanding the statement or the conditions of the problems and, consequently, in the way in which they could apply the contents they were currently working on. Also, there were still some groups with difficulties in expressing their mathematical reasoning in writing, driven by the need to construct the poster. This formal artefact, which would later be object of scrutiny, aroused a greater concern in the students regarding written communication. Despite showing greater attention to what would be registered, some groups still showed mistakes or shortcomings, such as: the absence of measurement units; the calculation of areas of figures that were not clearly identified in the image presented; or the writing of wrong equalities by associating calculations that should be independent.

Steps 3 and 4: Presentation and Observation of Posters and Analysis and Elaboration of Comments

For a better perspective of the different posters, they were displayed in the corridor near the classroom, facilitating the students' movement in the surrounding space. Sometime was provided for them to conveniently analyze each poster, allowing a reflection about the work developed by their colleagues, so that, at a later stage, each student could make their individual comments on the observed solutions. Figure 9 illustrates the moment of the formulation of comments.

Figure 9. Elaboration of Comments during the GW

Analyzing the feedback formulated to the poster of each group in terms of its focus, we can observe in Table 4 that there was some diversity in the subcategories, with the emergence of feedback centered on the student, the task, the process, the result and on self-regulation, although comments focused on the students were not used on the second task. It is also noticeable that not all groups received the same type of feedback, concerning its focus.

Table 4. Summary of the Type of Focus of the Written Feedback

Tasks	**Focus of the written feedback**	**Groups**					
		1	**2**	**3**	**4**	**5**	**6**
1	Student		x	x	x		
	Task	x	x	x		x	
	Process			x	x		x
	Result	x		x		x	x
	Self-regulation	x		x			x
2	Student						
	Task		x	x	x	x	
	Process	x	x	x	x		x
	Result	x		x	x	x	x
	Self-regulation	x		x			x

The comments envisaged different purposes according to the focus, namely:

the feedback focused on the students was intended to praise aspects of the work carried out, such as the groups' involvement, their effort, for example, in the presentation of diversified solutions or the careful preparation of the poster;

the feedback focused on the task (see Figure 10) aimed only at its correction, mentioning if the task was adequately solved or not;

the feedback focused on the process (see Figure 11) tried to call the attention of the groups to the correction/clarification of the strategies and the calculations used, as well as underlying the need to contextualize some numbers or operations;

in turn, the comments focused on the result (see Figure 12) alerted the students to the correction of the answers presented;

finally, the comments focused on the self-regulation of learning alerted to issues that would help improve students' performance (see Figure 13 and Figure 14).

In the domain category, the comments made by the students focused mainly on aspects related to the mathematical content and the clarity and correctness of the language used. In the case of groups 4, 5 and 6, there were also comments made regarding questions of organization and poster presentation. Regarding the syntactic structure and the extension of the comments made, the feedback presented was entirely of short extension and had an affirmative syntactic structure. This situation was somewhat to be expected; on the one hand, the comments were written in the limited space of a post-it and, on the other hand, these students had no experience so far in the formulation of written feedback, being more natural to write observed facts through a statement than to formulate dialogic questions inducing reflection.

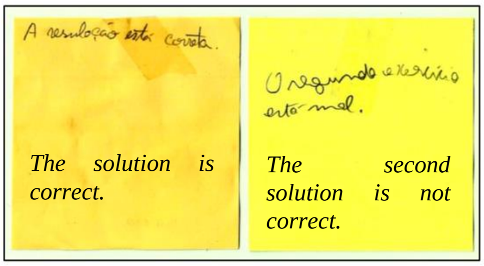

Figure 10. Examples of Feedback focused on the Task

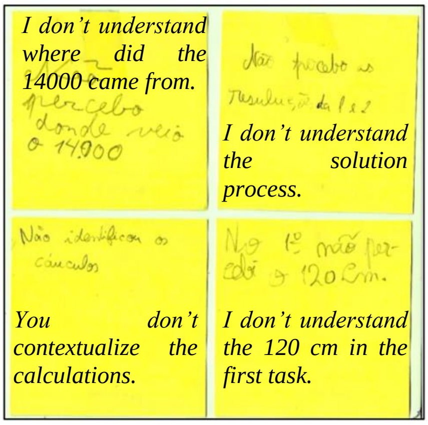

Figure 11. Examples of Feedback focused on the Process

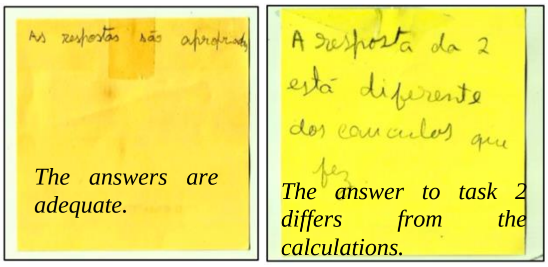

Figure 12. Examples of Feedback focused on the Result

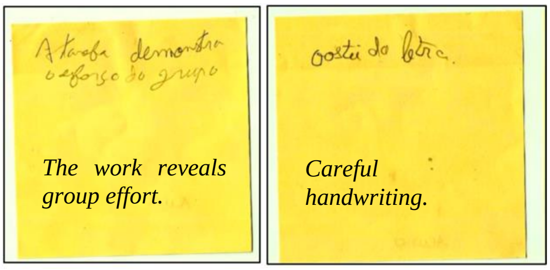

Figure 10. Examples of Feedback focused on the Student

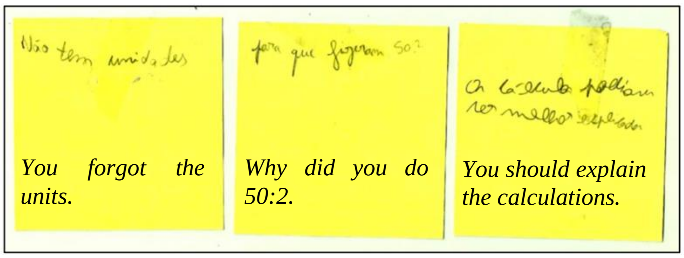

Figure 14. Examples of Feedback focused on Self-regulation

Steps 5 and 6: Group and Collective Discussions

In the group discussion, the students started by analyzing the comments made to their solutions. It was possible to infer that they saw the comments as a way to improve the work developed, highlighting their importance for a better performance in solving the tasks: "Someone did not understand how we solved the first task. We thought the solution was complete, but it wasn't. We should have explained it better, but it's hard to explain it in writing" (JB – Group 6); "We were able to better understand some of the mistakes we made" (LA - Group 3). After analyzing the comments, the groups proceeded to the collective discussion, presenting the posters to clarify the whole class about each solution and, on the other hand, react to the comments made to their work. During this step of the GW, the posters were disposed side by side on the classroom board so that everyone could access their content.

The students of each group started by explaining how they thought to solve each task, reacting simultaneously to the comments on the post-its that, in several situations, were a lever to reanalyze their work. Group 1 took the opportunity to mention that, in task 1, despite having achieved a correct answer, they forgot to register all the calculated areas, presenting only the areas of two triangles when there were three. As for task 2, they were alerted by the teacher about the writing of wrong equalities, a fact that had not appeared in the feedback received and that the teacher saw as an opportunity to alert the whole class, as other students committed the same error.

Group 2 emphasized the visual strategy they used in solving the first task, to show a different reasoning from that already presented by the previous group and also that "a problem can be solved in different ways". They had the opportunity to explain their solution in more detail, since not all colleagues understood how they thought, recognizing that "only with the comments did they realize that some colleagues did not understand what they wanted to say in the solution". In task 2, they acknowledged having made a mistake in writing the answer, 1800 instead of 18000 m^2.

Group 3 was called on the fact that it was necessary to better contextualize the calculations presented in task 1. In turn, in task 2 they made some mistakes, having started the explanation by referring the difficulties they felt: "Our reasoning was wrong because we thought that the area of the rectangle was equal to the area of the parallelogram"; "Well, and we only realized it when we saw the solutions of other groups and the comments they left us". For the students in this group, the comments received were effective to see if they had solved the tasks correctly or not, however, they considered that the feedback was not enough to understand specifically how they could improve. However, the solutions of the other groups became an asset that allowed them to better understand their mistakes.

Group 4 wanted to present the solution of the first task because they considered it to be original and correct, a "visual" solution, but the same did not happen with the second task. This group made mistakes that they admitted having understood after analyzing the post-its and the solutions of colleagues: "We thought we could divide the figure like this. But we already realized that we couldn't!"; "The truth is that there is nothing to guarantee that our decompositions are right".

Group 5 presented strategies similar to those of previous groups and, therefore, they did not need to clarify their reasoning. However, they took the opportunity to mention that the comments they received served to understand that they should have presented a better contextualization of the calculations, being, therefore, an aspect to improve in their solution.

The solutions presented by Group 6 were considered different and original when compared to the ones presented by the other working groups, since they chose to use strategies with a predominance of the visual approach. However, the students recognized that the strategy presented in task 1 was not the most adequate: "We did a lot of unnecessary calculations and we only realized this when we read the comments and thought about it".

Conclusion

Based on the collected data, supported by the theoretical framework, the main conclusions of the study are presented and organized according to the research questions initially formulated.

Considering the presented results, we can consider that the different groups globally had a good performance throughout the GW. In fact, of the six groups, none presented an incorrect solution in task 1 and only two groups presented an incorrect solution in task 2. However, even though there was a high percentage of success in solving the proposed tasks, there are difficulties to highlight. One of the most frequent is related to the understanding of the task statement. Difficulties of this nature would already be expected in some of the tasks, since we tried to use proposals with different levels of cognitive demand, low and high (Stein & Smith, 2011; Vale & Barbosa, 2021). In turn, in some of the solutions presented, it was still possible to identify difficulties related to the contents of the topic Areas of plane figures, mainly in terms of understanding and applying the formulas to calculate the area of some figures. This situation is supported by the literature. For example, Huang and Witz (2011), who reported difficulties of this nature on the part of students, explained that they are due to the transition from using a counting strategy, used in the yearly years, to the conceptual understanding of the area formula that is not yet consolidated. We must also highlight the difficulty that the students felt in the application of measurement units associated with each proposed task. According to Lavrador (2010) and Huang and Witz (2013), this difficulty is related to the poor understanding that students may have about the concept of area, leading them to misconceptions that translate into poor application of knowledge.

To solve the tasks, the students mobilized their knowledge about the areas of plane figures and presented some diversity in the strategies used. As would be expected, analytical strategies were predominant, with frequent use of algebraic, numerical and verbal representations. However, other strategies were used, of mixed and visual nature, with the figures playing a fundamental role in the students' reasoning (Borromeo-Ferri, 2012; Presmeg, 2014; Vale et al., 2018). Students benefited from participating in a GW, having the opportunity to come into contact with visual solutions, which often involved a dynamic perspective (Huang & Witz, 2011; Presmeg, 2014; Vale et al., 2018) as a complement to approaches of analytical nature. In fact, it was possible to understand that the GW allowed students to experience solving tasks with multiple solutions and with different levels of cognitive demand, as proposed by the NCTM (2014).

The moment of elaboration of comments in the GW allowed to identify comments with distinct characteristics. Regarding the focus, the feedback showed a greater focus on the task, on the process and on the result. In fact, the comments formulated followed the conjectures of Hattie and Timperley (2007) who state that effective feedback must maintain its focus on the task and on the process. According to these authors, these types of feedback are very promising for improving the development and understanding of the proposed task. On the other hand, Pereira (2008) and Nelson and Schumn (2009) mention the importance of feedback with an impact on the result, since it leads to reflection on the work carried out, allowing it to be improved. Regarding the domain, students focused more on the mathematical content than on the clarity and correctness of the language or on the presentation and organization of the poster. Finally, it should be noted that, as for syntactic structure, students only presented affirmative feedback, a fact proven by Bruno (2006) who mentioned that it is a simpler way of attributing feedback, although it does not promote reflection as much as feedback of interrogative type. It is also worth noting that regarding extension, students always presented short feedback, promoting an easy reading and clarification on the part of the receiver (Santos & Semana, 2015). These results would somehow be expected taking into account the type of feedback given in a GW, short and immediate, on a post-it.

The participation in a GW allowed, above all, an evolution at a social, physical and cognitive level, making students increase their repertoire of strategies, due to its presentation. The written feedback constituted as a means of promoting the refinement of the strategies used, helping students improve the work presented as preconized by some authors (Barbosa & Vale, 2021; Katsberg et al., 2020). Regarding the feedback given, the students understood that, if they formulated comments at the domain of the mathematical content, they would help the receiver correct and/or improve the work presented. In fact, the improvement of the work only occurred when, on the one hand, the feedback was based on the emerging mathematical content and, on the other hand, it had an impact, on the task, on the process and on self-regulation of learning, as was stated by Hattie and Timperley (2007). However, the existence of several comments with an impact on the result was also notorious. As Pereira (2008) mentions, although this type of feedback does not lead the student to reflect directly on the presented solution, it led some groups to question these types of comments, leading them to a small reflection in order to verify if, in fact, the result presented was correct or incorrect. In turn, the way the comments were written was also a determining factor for a better understanding of the message they wanted to convey, that is, despite all the comments being presented with an affirmative syntactic structure and of short extension, it was important that the students provided the clues and/or suggestions for improvement as clearly as possible, so that the receiver could proceed with its rectification. It is important to note that, already in the literature review, Brookhart (2008) and, later, Terroso et al. (2019), verified the importance of clarity and simplicity of the transmitted information, being essential to adapt the language, vocabulary and the content of what is communicated to the recipient of the feedback.

This study made it possible to conclude that feedback can influence students' performance, however it should have specific characteristics so that it translates into significant information to improve the work carried out. It was also noticeable that the feedback that promotes self-correction is the one that focuses on the task, on the process and on the result and that, above all, focuses on the mathematical content involved. Furthermore, it must be simple and clear, in order to indicate concisely what it intends to translate. It is also important to point out that this study also showed that, with practice, students can improve the feedback given and, therefore, a systematic practice at this level is relevant in order to translate into better results in terms of learning.

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