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# The Influence of Dynamic Representations in Mobile Applications on Students' Learning Achievements in Solving Multiple Integrals

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## Abstract

In this paper, we describe the influence of the implementation of methodology for teaching and learning multiple integrals based on the constructivist approach and the interconnectedness of multiple representations (with an emphasis on dynamic representations) for different mathematical objects using GeoGebra's applications Graphing Calculator and 3D Calculator. The empirical research was conducted at the University of Kragujevac, Serbia, with two groups (the experimental and the control groups) of second-year students. During the learning process of the experimental group, students created digital materials to visualize the domain of the integration for the concrete tasks, to set the integration bounds, to recognize in which situation they needed to introduce the switch of variables, and all that to solve double or triple integral problems. Students in the control group, on the other hand, didn't use technology of any kind, including the programs Graphing Calculator and 3D Calculator. The findings of the empirical study showed that students' use of these tools to create digital materials improved their academic achievement and their practical and theoretical knowledge of multiple integrals.

## Introduction

In the last few years, the education process has rapidly changed. New technology learning environment has been increasingly used for learning mathematics, in particular for calculus (Stevanović et al., 2021; Psotka, 2022; Mitrović et al., 2022; Runge et al., 2023). Since university students are the largest demographic of mobile users, the teaching and learning process in higher education can be improved by using modern technology, such as laptops, tablets, and smart phones (Crompton & Burke, 2018). The use of tablet computers by students facilitates teaching and learning process in higher education (Venema & Lodge, 2013). It is shown that the use of technology contributes to better achievements of these students in higher education (Devlin & McKay, 2016). In their research, Devlin, and McKay (2016), emphasize the benefits of the use of technology in teaching and learning, such as the possibility of creating electronic materials, which are available to students at any time.

Besides the mentioned, the use of technology in teaching and learning enables creating an environment appropriate for constructivist learning (Božić et al., 2019). Constructivism is a learning theory, based on the idea that student actively creates his knowledge using previous knowledge and experience (von Glasersfeld, 1995; Takači et al, 2015). The student creates the meaning of new information independently, not passively accepting the knowledge from the teacher, connecting them with his previous knowledge. This means that the existed students' knowledge is enriched and adopted to new experiences. During learning, the student must adapt his existing knowledge with the new experience, so that it becomes more complex (Takači et al, 2015).

The role of teacher in constructivism is very important because he is supposed to monitor and coordinate students' learning to help students to improve their understanding and interpretation. The teacher must take care of the previous experiences of students, the uncertainties and difficulties associated with new learning content, to improve the quality of knowledge. The teacher must stimulate students' curiosity and encourage discussions (Bhowmik, 2014; Al-Huneidi & Schreurs, 2012).

Previous research has shown that the use of different representations of the observed object has a positive influence on the students' constructivist learning (Božić et al, 2019). Representations are used in learning to help students to understand and to connect the contents. According to Hwang & Hu (2013), representation is the process of modeling concrete objects in the real world into abstract concepts. The purpose of representation is to support the transition between concrete and abstract thought. Representation of a particular object is being formed by comparing the properties of that object with the properties of another, from a previously known object (Font et al., 2007).

In the learning process, representations are used to help students to understand and to connect the contents. Many authors pointed out that there are two basic systems of representations. These are external representations, that are found in the student's surroundings, and which are being observed by the student, and internal (mental or psychological) representations, which are being constructed in the student's mind by himself (Hwang et al., 1998; Goldin & Shteingold, 2001; Kilpatrick et al., 2001; Nakahara, 2008). The representation makes sense only if it is observed together with other representations and when their interconnectedness is analyzed. The quality of interconnection of representations in the system is very important for the understanding of representations by an individual (Goldin & Shteingold, 2001; van der Meij, & de Jong, 2011). Multiple representations represent different representations offering the same information in different forms. They provide information about the observed object in different ways, so the observer can analyze its properties from different points of view (Ozgun-Koca, 2008). In mathematics they provide a convenient environment for abstracting and understanding key concepts (Borba & Confrey, 1996; Hwang & Hu, 2013).

The use of modern technology contributes to the improvement of multiple representations. The use of technology has significantly contributed to the development of visualization through the improvement of graphical representation. There are software packages that allow simultaneous display of two or three representations of the same object. Therefore, there are numerous research on the application of modern technology for the improvement of the quality of multiple representations (Rau et al., 2015; Sengoren, 2014; Sever & Yerushalmy, 1991; Ozgun-Koca, 2008; Tall, 2003) of mathematical concepts.

Dynamic software packages allow a change in one representation to cause simultaneous change in other representations of the same object (Hwang & Hu, 2013). In their research, Ermete et al. (2010) emphasize the use of technology for dynamically linking multiple representations, to improve connections between mathematical concepts, made by students. It is shown that the use of dynamic software packages, such as Cabri, GeoGebra and Geometer's Sketchpad, contributes to improving pre-service teachers' competences in terms of geometry (Stols, 2012).

The improvement of mathematical representations, which can be seen from the above, was especially pronounced by the introduction of manipulative representation (Nakahara, 2008). This representation usually contains the elements of different representations, such as graphical or algebraic, and it was created by dynamically linking of these representations, supported with the appropriate dynamic software. Dynamically linking of multiple representations and improving dynamic representations became an interesting topic for contemporary researchers (Arzarello et al., 2012; Ermete et al., 2010; Guven, 2012; Lisarelli, 2017).

GeoGebra software packages enable dynamic representations i.e., dynamically linking of multiple representations. The most important characteristic of this software is its possibility of simultaneously observing the algebraic and the graphical representation, so that changing of one of these representations results in the instant updating of the other one. The use of GeoGebra as a teaching aid to improve multiple representations is also a popular topic in contemporary research. In that research, benefits of using GeoGebra in learning mathematics, particularly calculus, are analyzed (Abu Bakar et al., 2010; Arzarello et al., 2012; Doruk et al., 2013; Takači et al., 2015).

Moreover, in addition to GeoGebra software, researchers also use GeoGebra applications such as GeoGebra 3D Calculator in mathematics classes in higher education (Funes et al., 2021). Another graphing calculator, the Desmos Graphing Calculator proved to be an effective learning medium (Biladina et al., 2022). In the last research, authors (Biladina et al., 2022) claimed that the Desmos Graphing Calculator application assisted learning in an effective way which reflected positively on student learning outcomes.

In higher education, multidimensional calculus is being studied. Multiple integrals are one of the most important concepts within multidimensional calculus. The multiple integrals and their application have significant importance in different scientific disciplines, such as engineering (Aguilera et al., 2007). However, this content usually causes a lot of difficulties for the students (Kashefi et al., 2010). Most of these difficulties are related to solving the multiple integral problems, which requires sketching 3D objects, i.e., visualization and connecting algebraic and graphical representation (Mahir, 2009). Previous research has shown that improving visualization contributes to the better achievements of the students, in terms of the integrals (Sevimli et al, 2012).

To achieve better visualization, different kinds of software are being used in teaching and learning the multiple integrals. Examining the influence of the software application on the students' achievements in learning multiple integrals was the topic of several research, and the interest in research in this area has especially grown in recent years (Gimenez et al., 2022; Henriques, 2006; Milenković et al., 2020; Xamroev, 2022, Zhou & Zeng, 2022). The use of multiple representations, within the dynamic software environment, proved to have a positive influence on the students' achievements in learning double integrals (Milenkovic et al., 2020). Gimenez et al (2022) point out that the students' multiple integrals – based activities, within the Wolfram Mathematica software environment, besides the improving students' achievements in this area, also have the positive influence on the students' motivation in solving mathematical – based problems in engineering. Previous research suggested that the student's multiple integrals learning process qualitative analysis should be conducted more in depth (Milenković et al., 2020).

The research conducted during the first semester of academic year 2021/2022 included a total of 72 second-year students of bachelor academic studies at the Faculty of Engineering, University of Kragujevac, Serbia, majoring in mechanical engineering, within the Mathematics 3 course. The mathematical education of students during their first year of basic academic studies did not differ in the form of teaching, teaching methods, teaching contents and their scope, nor in the number of teaching hours of lectures and exercises classes. The groups (control and experimental) of students were not homogeneous (the groups consisted of students of different previous achievements in Mathematics 1 and Mathematics 2).

**Research Question**

As many studies have shown a positive impact of using mobile devices in classroom on students' knowledge and understanding, and having in mind the benefits of planned usage of the applications enriched with possibility of multiple representations (specially dynamic representations) of mathematical objects, with constructivist learning approach, on students' success in dealing with teaching materials referred to calculus, we designed the experimental approach for teaching and learning multiple integrals.

In our research, the main question is:

Does the planned usage of mobile applications by students for visualization of the functions and other geometric objects in plane and in space contribute to better students' achievements regarding multiple integrals?

We expected that the students would learn better with the materials created by themselves in Graphing Calculator and 3D Calculator with better success as compared to students who didn't use these applications while learning multiple integrals. Thus, the hypotheses for the research question are:

H1) The usage of GeoGebra mobile applications for creating materials that represent part of a plane or space leads to better understanding of those sets of points and defining them adequately, i.e., setting boundaries for the appropriate variables.

H2) The usage of GeoGebra mobile applications for creating materials regarding multiple integrals tasks leads to better students' success in solving those tasks.

H3) The usage of GeoGebra mobile applications for creating materials that represent part of plane or space defined with the variables introduced after the appropriate switch of variables leads to better understanding of those sets of points and defining them i.e., setting boundaries for the introduced variables after the switch of variables.

## Method

### Sample (Participants)

In our research, we considered two groups of second year students from the Faculty of Engineering, University of Kragujevac, Serbia.

- In the control group there were 37 students from the study program Mechanical engineering.
- In the experimental group there were 35 students from the study program Mechanical engineering.

### Design of the Study

The teacher pointed out to the students from the experimental group that the Graphing Calculator application could be used to draw graphs of functions, for solving equations and systems of equations with graphical method, as well as for graphical presentation of mathematical objects, while the 3D Calculator application can be used to draw graphs of multivariable functions (in space), then to sketch the graphs of surfaces, to represent geometric objects in space and solve problems from stereometrics.

Having in mind that students, during their mathematical education at the university level, did not deal too much with the analytical geometry in the plane and space, and that the method of learning and teaching mathematics in the previous two courses of mathematics was much more analytical, and not visual, the teacher chose those two applications for visualization the mathematical objects to solve multiple integrals. Students from the experimental group were told to install applications Graphing Calculator and 3D Calculator on their mobile devices. Students could choose devices (mobile phones or laptops) according to their abilities, and they were informed that they should bring their devices to the exercise classes.

Also, the teacher told the students that they will create teaching materials that follow the appropriate tasks by themselves, and that by creating an account on the GeoGebra website, the given materials can be used later during learning process and practice, as well as that they can further refine them. At the beginning of the first lesson of multiple integrals, the teacher determined whether all students had installed the applications and went through certain details, certain technical aspects of the given applications. When it comes to the language used in the application, students were able to choose which language they wanted to use.

Before making concrete examples, the teacher introduced the students to the form and method of further work - after the teacher gives the students an appropriate example, i.e. task, students independently, possibly by working in pairs (if it is necessary for technical reasons), using applications Graphing Calculator and 3D Calculator draw graphs of functions of one variable, functions of two variables, lines, curves and surfaces, depending on the concrete task or the example. The teacher introduced the students to the dual nature of two applications, more precisely that by writing equations (or inequalities) of mathematical objects in the appropriate box (algebraic representation), graphical representations of given mathematical objects are obtained.

The process of solving the tasks went so that the teacher would read the task i.e., instruct the students in the task request, then allow the students to create the appropriate digital content for the task, then to cross out the image in their notebooks, to determine the domain for integration, to determine the integration bounds and after that they would start calculating the multiple integral manually. Drawing a suitable picture in notebooks, although it may be redundant at first impression, was considered desirable because in that manner students would practice creating graphical representation of appropriate mathematical objects in this way, and students did not have the opportunity to use mobile devices during the exam.

Since the students were introduced to the concept of double integral in lectures in Mathematics 3, they solved double integrals in exercise classes and used them to determine the area of a part of a plane, i.e., the volume of a part of space bordered by graphs of functions of two variables and surfaces. For the tasks in which students should determine the volume of the body formed by the intersection of three-dimensional geometric objects, students made two graphs, one in 3D Calculator to analyze the given multivariable functions, surfaces and planes, and then the other one in Graphing Calculator, after determining the projection of the body on the projection plane, with the aim of precisely determining the boundaries of the two variables, $x$ and $y$.

During the experimental work, students created digital contents to understand the meaning and properties of variables after the switch of variables using polar coordinates (in Graphing Calculator), or after the switch of variables using cylindrical and spherical coordinates (in 3D Calculator). With the option of leaving a trace of a point in the plane (or space) and with the help of sliders, students created ordered pairs (triplets) of points in accordance with the corresponding changes of variables to understand how changing the value of one of the two (three) variables affects the position of the point in the plane (space). Such materials are shown in Figures 1, 2, and 3, while materials created for some of the specific tasks are shown in Figures 4 and 5.

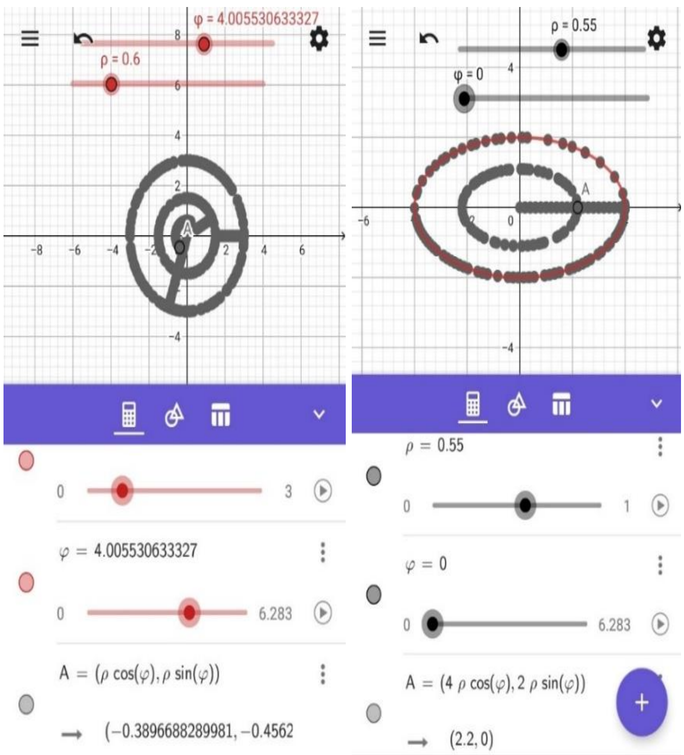


Figure 1. Switching from Cartesian to Polar Coordinates (for Circle and for the Ellipse)

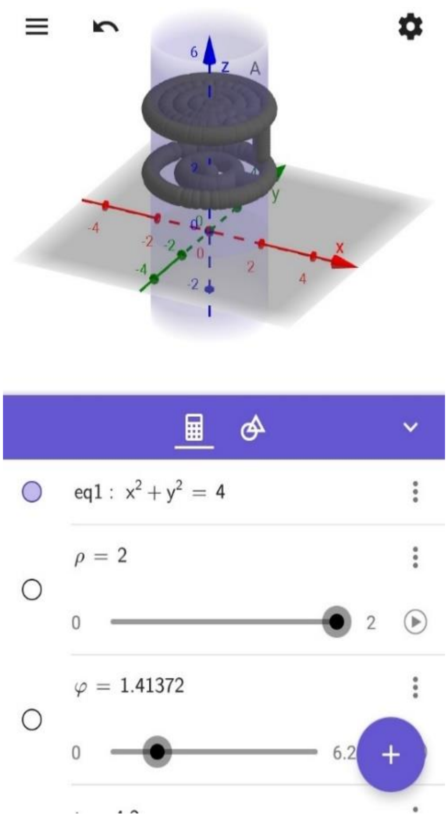

Figure 2. Switching from Cartesian to Cylindrical Coordinates

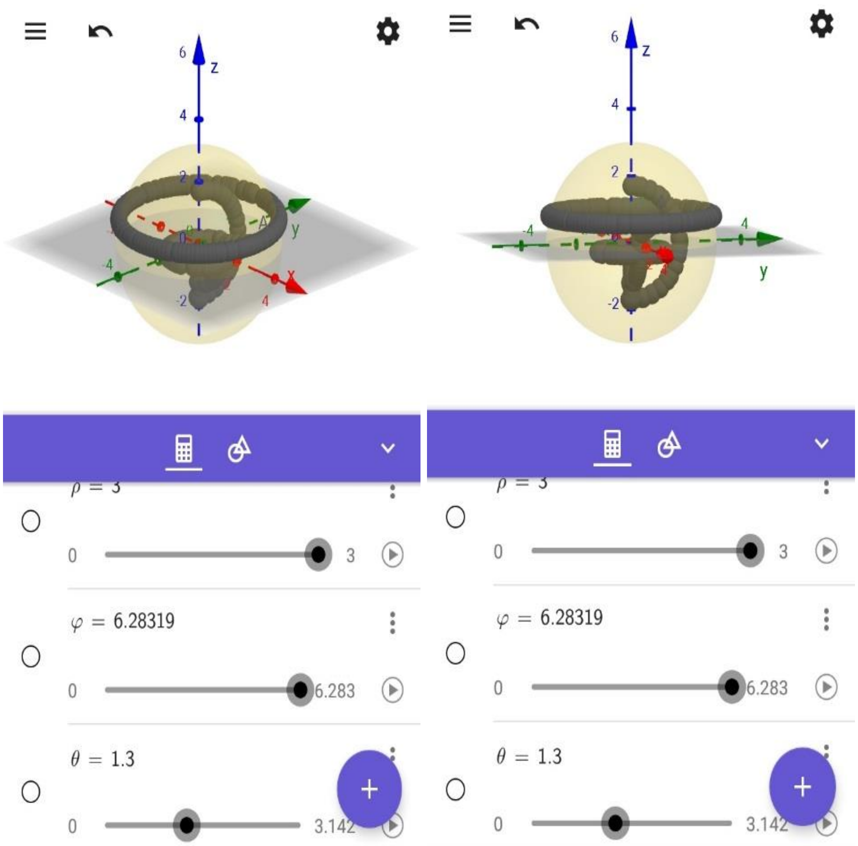

Figure 3. Switching from Cartesian to Spherical Coordinates

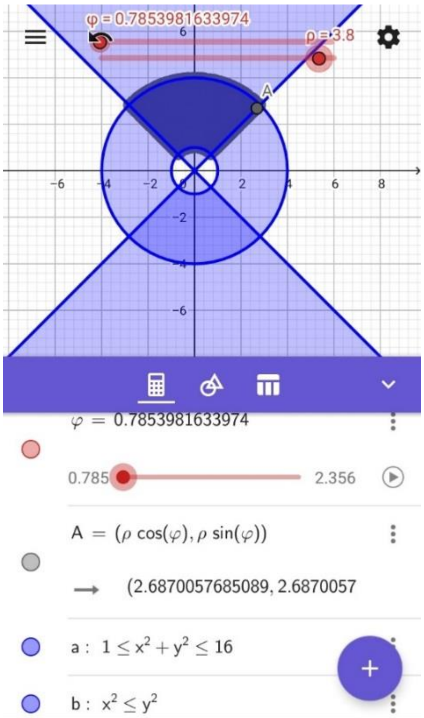


Figure 4. Graph for the Task in which the Integration Region is Defined with a Circular Ring and Two Lines

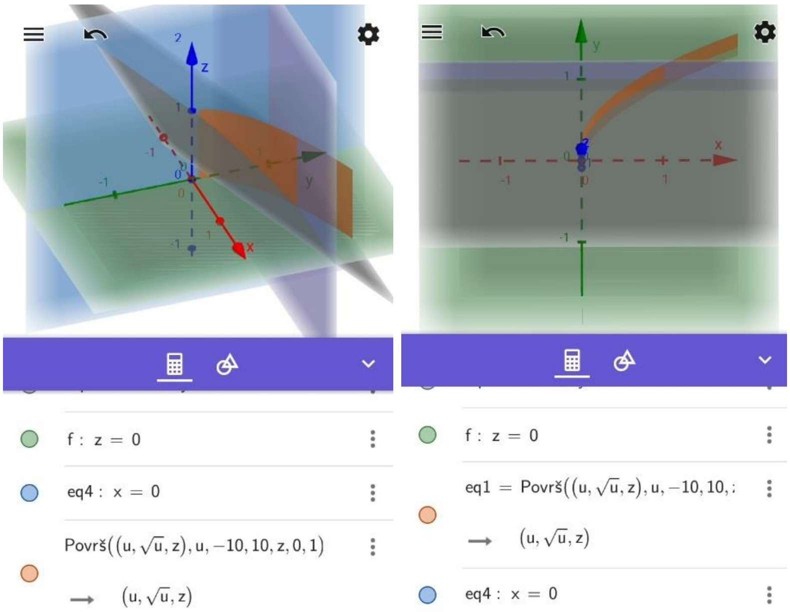


Figure 5. Graphs for the Task in which the Integration Region is Defined with Cylinder $y = \sqrt{x}$ and Planes $z = 1 - y,\ y = 1,\ z = 0, x = 0$

During the students' work in the classes, the students had the opportunity to save the materials in the appropriate format (they were instructed to do so) or take pictures of them so that they could analyze them later.The process of teaching and learning multiple integrals with the students of the control group took place without the use of mobile devices for visualizing teaching content. Namely, the students of the control group attached pictures drawn manually in their notebooks for the concrete assignments, while the calculation process of multiple integrals was carried out manually, in both groups.

The teaching process of double and triple integrals (in exercise classes) were realized during nine classes (four and a half terms) with both the control and experimental group.

## Results

After forming the experimental and control group, both groups of students solved the pre-test. The pre-test contained four tasks, and the students had 30 minutes to complete the test. Students could not use any help in solving tasks on the pre-test (computers or telephones). Pre-test tasks were chosen so that on the students' work, their procedural and theoretical knowledge of a definite integral was tested to evaluate the necessary prior knowledge to solve multiple integrals. The maximum number of points on the pre-test was 20.

In the pre-test there were not statistically significant differences between the groups tested, the experimental and control group at the level of significance of 0.05. The results of the statistical analysis for the pre-test are given in Table 1.

Table 1. Statistical Results of the Pre-Test

| Group | Number of students | Mean | Std. deviation | Student's t-test | | |
|---|---|---|---|---|---|---|
| | | | | $df$ | $t$ | $p$ (2-tailed) |
| Experimental | 35 | 11.09 | 3.89 | 70 | $-0.6$ | 0.55 |
| Control | 37 | 10.51 | 4.16 | | | |

The test consisted of ten tasks. The first five tasks were not directly related to the calculation of multiple integral. They were chosen to provide feedback on the level of students' knowledge of precise determination of areas in the plane, i.e. space, as well as redefining the appropriate areas after the switching from one coordinate system to another, i.e. mapping of the corresponding coordinate system into a rectangular coordinate system, because this is the one of most significant part of the concrete task solution for multiple integral problems. The second part of the test consisted of five tasks also (tasks 6 to 10) in which students were asked to calculate multiple integrals, or to use them to solve a certain problem. The maximum number of points that students could score on the test was 50. On the first, second and fifth task, students could score three points, on the third, fourth, sixth, seventh, on the eighth and ninth tasks, they could score six point (the 3a) and 3b) were evaluated with three points each, as well as the 4a) and 4b)), while the tenth task was evaluated with five points. There were no negative points. The average number of points per task for the students of the experimental and control group is given in Table 2.

Table 2. Average Number of Points of the Experimental and Control Group Students Per Task

| Group/Task | 1. | 2. | 3. | 4. | 5. | 6. | 7. | 8. | 9. | 10. |
|---|---|---|---|---|---|---|---|---|---|---|
| Control group | 1.59 | 1.65 | 3.46 | 2.73 | 1.59 | 3.62 | 2.68 | 2.65 | 2.38 | 2.81 |
| Experimental group | 2.00 | 1.71 | 4.00 | 4.03 | 2.09 | 3.69 | 4.23 | 3.46 | 3.54 | 3.69 |

Since the tasks can be divided into two groups - the tasks related to the solving of multiple integrals (with prior determination of the domain of integration, and determination of the boundaries of variables) and tasks that give insight into students achievements in the determination of the sets of points in 2D and 3D and adequate transformation of variables , i.e. on the tasks that represent a very important part of the task related to multiple integrals, the analysis of the students' achievements was logically imposed. On that basis we analyze the students' achievement in solving the first five tasks, and then we analyze the students' achievement in solving the second five tasks. As the Kolmogorov-Smirnov test for normality for the variable representing the total number of points that students from the experimental and control group scored while solving the first five tasks, showed that the values within the sample do not follow the normal distribution ($p = 0.03$ in the control group, $p = 0.03$ in the experimental group) at a significance level of $0.05$, we took adequate (Mann-Whitney U) test as a non-parametric statistical technique to analyze differences between distribution of number of points among experimental and control groups of students. The distribution of the number of points achieved by solving the first five tasks on the test of the students from the experimental and control groups is presented in Figure 6.

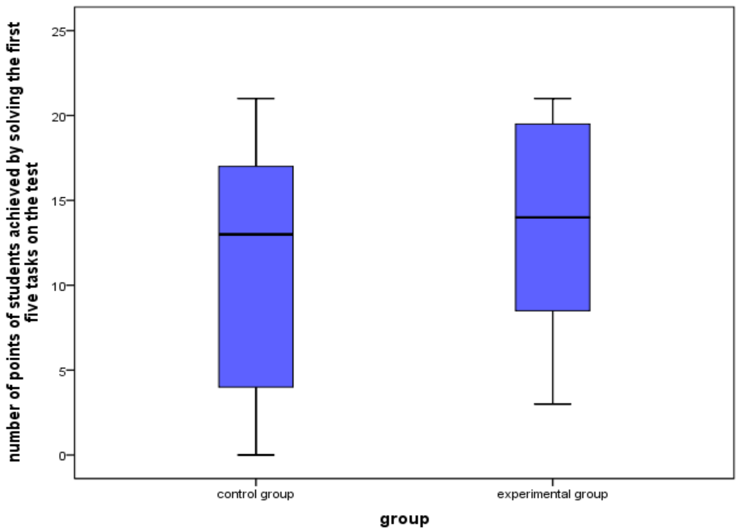


Figure 6. Distribution of the Number of Points by Solving the First Five Tasks on the Test

Based on the values given in Table 3, it can be concluded that there is no statistically significant difference ($p = 0.069$) in students' achievements for the students from the control group and students' achievements for the students from the experimental group in solving the first five tasks on the test, with a significance level of $0.05$ (although the difference is statistically significant at level of $0.1$).

Table 3. Statistical Results of the Total Number of Points Achieved by Students while Solving the First Five Tasks

| Group | Number of students | Mean | Median | Mean rank | Sum of ranks | Man-Whitney U test | |
|---|---|---|---|---|---|---|---|
| | | | | | | Z | $p$ (2-tailed) |
| Experimental | 35 | 13.83 | 14 | 41.10 | 1438.5 | $-1.82$ | 0.069 |
| Control | 37 | 11.19 | 13 | 32.15 | 1189.5 | | |

After that, the potential differences in student success in solving multiple integrals tasks were examined. The students' achievements for the students from the control group and from the experimental group are given in Table 4, while the distribution of the total number of points achieved by students from the experimental group and the students from the control group in solving the second five tasks on the test is presented in the Figure 7.

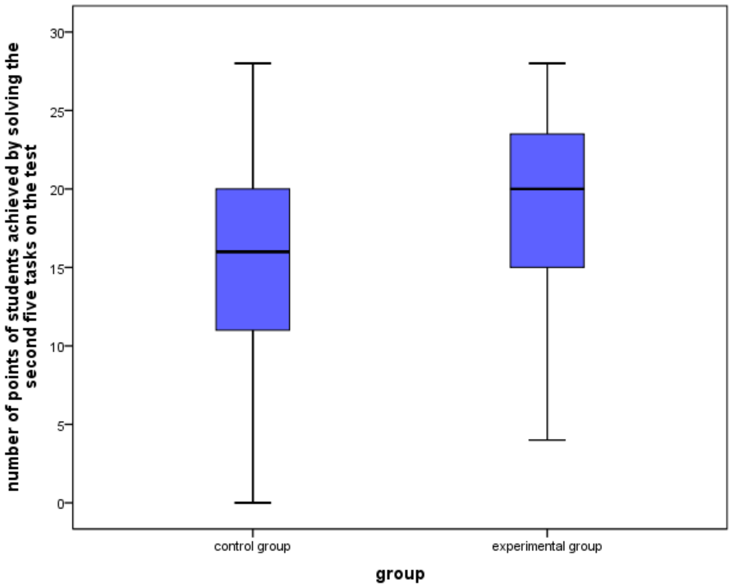


Figure 7. Distribution of the Number of Points by Solving the Second Five Tasks on the Test

Based on the Kolmogorov-Smirnov test, the variable representing the total number of points students scored while solving the second five tasks had a normal distribution in both the control ($p = 0.645$) and the experimental group ($p = 0.112$). Therefore, we examined potential differences in students' achievement in solving multiple integrals using the Student' t-test. Based on the values obtained on the appropriate analysis, we can conclude that there is statistically significant difference in the students' achievements for the students from the control group and the students from the experimental group, in solving double and triple integrals ($p = 0.016, \eta\text{^}2 = 0.08$) at the level of significance $0.05$, in favor of students of the experimental group.

Table 4. Statistical Results of the Total Number of Points Achieved by Students while Solving the Second Five Tasks

| Group | Number of students | Mean | Median | Std. deviation | Student' t-test | | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | $df$ | $t$ | $p$ (2-tailed) | $\eta^2$ |
| Experimental | 35 | 18.60 | 20 | 6.02 | 70 | $-2.48$ | 0.016 | 0.08 |
| Control | 37 | 14.84 | 16 | 6.91 | | | | |

Based on the obtained results, it can be assumed that, in most cases, students who have successfully, graphically, and then analytically determined the domain of integration, determined the boundaries of variables, introduced switch of the variables (if necessary) and determined the boundaries of those variables, did not have many problems in carrying out the computational part of the task. Thus, students who did not master determining the domain of integration and setting the boundaries of variables, had nearly no chance in further solving tasks (from sixth to tenth), which once again indicates the importance of the first part of solving the task, the part on which we influenced students from the experimental group by using mobile applications Graphing Calculator and 3D Calculator for visualization of the mathematical concepts.

Although the differences in the students' success in solving problems related to multiple integrals were statistically confirmed, we were especially interested in the influence of application of the Graphing Calculator and 3D Calculator applications for introducing switch of variables and mapping the appropriate coordinate system into a Cartesian coordinate system. Therefore, we analyzed the differences in students' achievements for solving the tasks in which it was necessary to introduce the switch of variables using polar coordinates, cylindrical coordinates, or spherical coordinates (third, fourth, fifth, seventh, ninth and tenth task), regardless of whether these switches were necessary to solve multiple integrals, or we wanted to determine the level of this kind of knowledge and students' skills. The distribution of the variable that represent the total number of points achieved by students from the control group and the students from the experimental group while solving tasks on the test in which it was necessary to introduce a switch of variables is presented in the Figure 8.

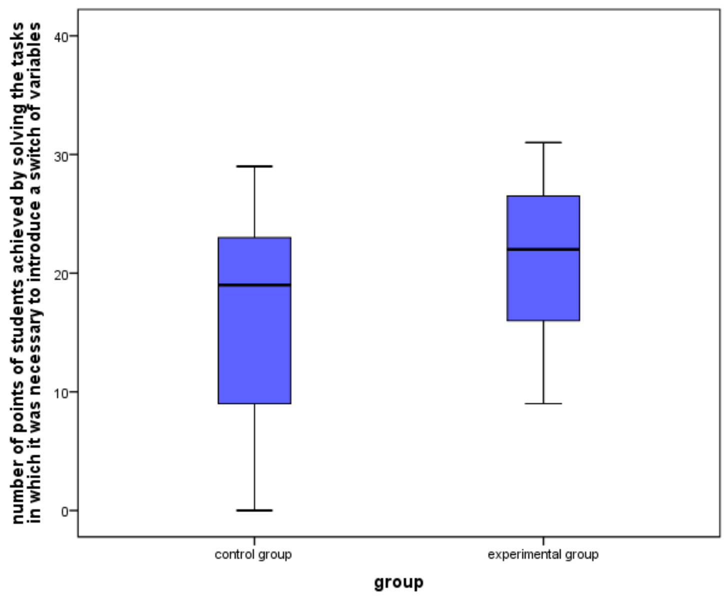


Figure 8. Distribution of the Number of Points by Solving the Tasks in which it was Necessary to Introduce a Switch of Variables

Based on the Kolmogorov-Smirnov test, the variable that represent the total number of points achieved by students from the control group and the students from the experimental group while solving tasks on the test in which it was necessary to introduce a switch of variables and thus observe appropriate mappings of the coordinate system in the plane, i.e. in space, in both groups of students, do not follow the normal distribution ($p = 0.012$ in the control group, $p = 0.038$ in the experimental group). For that reason, we compared the distributions of the given variable by using the Mann - Whitney U test.

Table 5. Statistical Results of the Total Number of Points Achieved by Students while Solving the Tasks in which it was Necessary to Introduce a Switch of Variables

| Group | Number of students | Mean | Median | Mean rank | Sum of ranks | Man-Whitney U test | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | Z | $p$ (2-tailed) | $r$ |
| Experimental | 35 | 21.57 | 22 | 43.47 | 1521.5 | $-2.75$ | 0.006 | 0.32 |
| Control | 37 | 16.24 | 19 | 29.91 | 1106.5 | | | |

Based on the obtained values (Table 5), we conclude that the difference in students' knowledge and understanding the tasks in which it is necessary to introduce a switch of variables, to choose correct variables for the switch, interpret appropriate variables and manipulate with given variables is statistically significant ($p = 0.006$, $r = 0.32$ ) at the level of significance $0.05$, in favor of the experimental group.

Finally, we analyzed the differences in the overall achievements of students who learned about multiple integrals in the traditional way (control group), and the students who used mobile applications (experimental group) in the process of learning and practicing the procedures of solving multiple integrals, especially in determining the area of integration and integration bounds. The distribution of the variable that represents the total number of points that students from the experimental and students from the control groups achieved on the test is shown in Figure 9.

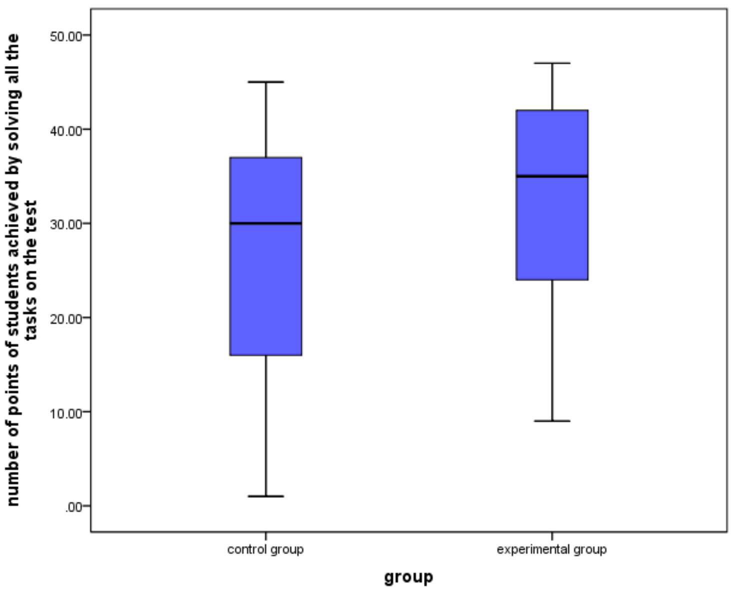


Figure 9. Distribution of the Number of Points by Solving the Tasks on the Test

Based on the Kolmogorov - Smirnov test, we found that the variable representing the total number of points that students achieved on the test does not follow the normal distribution in the control group ($p = 0.017$) and in the experimental group ($p = 0.016$). We compared the distribution of the total number of points in the control and experimental group by applying the appropriate, non – parametric, Mann - Whitney U test.

Table 6. Statistical Results of the Total Number of Points Achieved by Students while Solving the Test Tasks

| Group | Number of students | Mean | Median | Mean rank | Sum of ranks | Man-Whitney U test | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | Z | $p$ (2-tailed) | $r$ |
| Experimental | 35 | 32.49 | 34 | 42.20 | 1477 | $-2.25$ | 0.025 | 0.26 |
| Control | 37 | 26.03 | 30 | 31.11 | 1151 | | | |

Based on the values given in Table 6, we can conclude that there is statistically significant difference in students achievements ($p = 0.025$) in solving the tasks that refers to determining the points in plane or in space, determining the boundaries for those sets of points, recognizing the need for the switch of the variables and solving the multiple integrals after precise determining the domain of integration and integration bounds, at a significance level of 0.05, in favor of the experimental group.

## Discussion

Our study aimed to demonstrate that students can succeed and achieve in visual, practical, and theoretical knowledge when it comes to multiple integrals contents when multiple representations environments are used, with an emphasis on the presence of dynamic representations of mathematical objects combined with constructivism. As constructivism is considered as an approach for teaching and learning based on the idea that learning is the result of "mental construction" of the learner (Bada & Olusegun, 2015), and that a constructive approach is associated with adequate representation of concepts and problems, i.e. that it is necessary to understand the problem, that the person who solves the problem imagines (creates in his mind) objects and relations that correspond to objects and relations in the external representation of the problem, and that the combination of constructivist theory of learning and application of technology leads to best application software tools in order to facilitate the implementation of teaching (Rakes et al., 2006), we came up with the idea to design and implement a methodological approach that would be based on constructivism, the application of multiple representations of mathematical concepts and the implementation of mobile applications in the teaching and learning process. Since constructivist theorists gave teachers instructions which they should follow during class and use the terminology "analyze", "predict", "create" during the dialogue with students (Brooks & Brooks, 1993), the idea was that all digital content that will be used for visualization of the mathematical concepts should be created by students, of course with the assistance and suggestions from the teacher. Thus, the students of the experimental group, as described in the paper, created appropriate digital dynamic materials for each task in Graphing Calculator and 3D Calculator applications and thus were able to analyze these graphical representations in a multi-representative environment.

Firstly, we noticed that there were no differences in the students' achievements when it came to their success in solving the first five tasks. Although, based on the results shown in Figure 6 and Table 3, it could be assumed that the difference in student success in solving the first five tasks was statistically significant (the arithmetic mean of the number of points in the experimental group was 13.83 points, while the arithmetic mean of the number of points in the control group was 11.19, where the student could score a maximum 21 points), the appropriate test did not confirm statistical differences with a significance level of 0.05 (but it did with a significance level of 0.1). On the other hand, the results shown in Figure 7 and Table 4 showed that students who used applications Graphing Calculator and 3D Calculator for learning double and triple integrals had better achievements in solving concrete multiple integral tasks which was reflected in higher number of points they achieved compared to students who did not have that opportunity. The results from Figures 8 and Table 5, along with the adequate statistical test show that there is statistically significant difference between the groups of students who used mobile applications and the group of students who didn't, while dealing with the problems in which it was necessary to comprehend the switch from one coordinate system to another after introducing the switch of variables. This is shown as for the tasks where the defining of the 2D or 3D area were requested by itself or whether that was necessary part of the task in which the domain of integration for multiple integral tasks were part of circle, cylinder, sphere or because of the integrand function. After analyzing the previously mentioned parts of the test in accordance with the characteristics of the tasks from the test, the differences between the students of the two groups at the level of the entire test were analyzed. The results (Figure 9 and Table 6) indicate that the difference between the groups of students who individually created and manipulated the materials, rotated those mathematical objects, and observed them from different perspectives, manipulated with the parameters in some of those materials and could use them after the classes for practicing the tasks and deepening their understanding and theoretical knowledge for the multiple integrals contents showed better success.

Having all the obtained results in mind, we can say that the students' learning achievements regarding multiple integral problems are significantly better when they use the applications Graphing Calculator and 3D Calculator in class (and after the classes) in order to create digital content that should help them in determining the domain of integration and boundaries for variables (even when the switch of variables is introduced for the multiple integral task) and solving the multiple integral tasks.

### Limitations and Future Work Directions

For this research, we made changes in the course in the teaching and learning of multiple integrals (not for the line integrals nor for the surface integrals). This also regards our further work directions. We plan to use the applications Graphing Calculator and 3D Calculator to assist students in dealing with line integrals and surface integrals in the future and to examine the impact of these changes on the teaching and learning process. Also, we did not focus on the variety in the students' learning characteristics or on the analysis of the students' learning process.

## Conclusion

The purpose of our research was to show that teaching and learning multiple integrals could be improved in accordance with constructivist theory and the implementation of mobile applications in the teaching and learning process. After the experimental work, the potential differences in the achievements of the students of the experimental group who learned about multiple integrals actively and the students of the control group who adopted the teaching contents in the traditional way were analyzed. The test consisted of two parts. The first part of the test consisted of tasks whose purpose was to examine the degree of students' understandings and their knowledge, skills, and abilities of precisely describing sets of points in plane and in space by choosing the dependent variable and the independent variable, as well as describing sets of points after introducing an appropriate switch of the variables and setting the boundaries for the new variables. The second part of the test consisted of five tasks to evaluate the students' knowledge and skills in solving multiple integrals: determining the domain of integration, correctly determining the boundaries of variables, i.e., a precise description of the mentioned area, and the procedure of calculating multiple integrals. Based on that, the potential differences in student achievements while solving the first five tasks and then the other five tasks were analyzed separately. First, there was no significant difference in solving the first five tasks, so we couldn't confirm our hypothesis H1 with a significance level of $0.05$ (but we can with a significance level of $0.1$, Table 3). Therefore, the differences in the students' achievement in solving multiple integrals between the students from the experimental group and the students from the control group were tested, and it turned out that the differences in the students' achievement were statistically significant in favor of the students of the experimental group, which supports the hypothesis H2.

Statistically significant differences in students' achievements in solving multiple integrals and the absence of those differences in students' achievements when it comes to determining and defining sets of points in a plane and determining the bounds of variables (with the same significance level $0.05$) could be explained by the fact that for solving multiple integrals, determining the area in which the integration of multiple integrals is performed, as well as correctly setting the boundaries of variables, is a very important part of the task, and without that, it is practically impossible to start with solving multiple integrals. These results confirm the impact of the application of visualization for multiple integrals in a dynamic environment. The success of students who independently created interactive materials in the Graphing Calculator for visualization of variables, after introducing switches for the variables using polar coordinates, variables $\rho$ and $\varphi$ after introducing switch for the variables using polar coordinates; variables $\rho, \varphi$ and $z$ after introducing switch for the variables using cylindrical coordinates; and variables $\rho, \varphi$ and $\theta$, after introducing switches for the variables using spherical coordinates, and the students' who didn't have this opportunity, was analyzed. Based on those results, we can conclude that the students of the experimental group understood the meaning of the given variables to a much greater extent, better noticed when it was desirable to introduce a switch for the variables, as well as which switch would be appropriate, and finally correctly mapped the appropriate coordinate systems. These arguments support our hypothesis H3.

At the end, by analyzing the total number of points achieved by students of the experimental group and students of the control group on the test, it can be concluded that there is a significant impact from the usage of mobile applications Graphing Calculator and 3D Calculator for the visualization (of the curves, lines, functions of one variable, multivariable functions, surfaces, and their intersections) and for defining bounds for the given variables, and on the students' achievements while solving different double and triple integrals in a way that allows better understanding of the relations of geometric objects and precisely defining the domain of the integration for the multiple integral tasks. Therefore, we can conclude that the usage of mobile applications by students for visualization of functions and other geometric objects in plane and in space contributes to better students' achievements in solving multiple integrals.

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