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Learning to Craft and Critically Evaluate Prompts: The Role of Generative AI (ChatGPT) in Enhancing Pre-service Mathematics Teachers' TPACK and Problem-Posing Skills

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Learning to Craft and Critically Evaluate Prompts: The Role of Generative AI (ChatGPT) in Enhancing Pre-service Mathematics Teachers' TPACK and Problem-Posing Skills

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Abstract

The use of generative AI (Chat GPT) for the process of posing mathematical problems was introduced to 15 pre-service teachers (henceforth referred to as "teachers") in a re-training program aimed at teaching advanced secondary school mathematics. After solving mathematical problems, they were given an assignment to pose and refine additional problems in accordance with the curriculum requirements by giving "prompts" to the AI environment and refining them as needed. In this paper, we focus on the qualitative analysis of three key stages experienced by the teachers: writing the initial prompt to request assistance from ChatGPT to improve the problem; the response provided by ChatGPT; and the teachers' reflections on how they utilized ChatGPT's recommendations to update and refine their problem. This analysis reveals that the chat's responses and the teachers' reactions to them made several key contributions to improving mathematical problems, particularly by enhancing the clarity of the problem and increasing its challenge and complexity. Additionally, it improved mathematical precision and showed how it could be linked to real-world applications, thereby increasing student engagement. The chat also encouraged inquiry-based thinking and provided guidance for helping struggling students. Overall, these contributions significantly enhanced the quality of the mathematical problems and improved their TPACK (theoretical, pedagogical, and content knowledge).

Introduction

Teaching mathematics, especially advanced high school mathematics requires teachers to be able to effectively pose problems to aid their students' understanding of the mathematical concepts, either by adapting existing problems (e.g., from the textbook, Cai & Jiang, 2017; Chan, 2023; Lim et al., 2023) or formulating new ones (Silver & Cai, 1996). For this, teachers must have appropriate didactic and pedagogical knowledge with respect to the different types of problems (e.g., how to implement the "what if not" strategy, Brown & Walter, 1983) and be aware of their students' knowledge base. Not all teachers are adept at posing problems that meet educational needs (Mallart et al., 2018) or show students the relevance of mathematics in their everyday lives, thus enhancing

student interest.

To mitigate this problem, it would be highly advantageous to offer appropriate scaffolding to present new perspectives and pedagogical ideas for posing problems. One route can be the use of artificial intelligence (AI), which has recently emerged and is yielding fresh opportunities in education (Baidoo-Anu & Ansah, 2023). Although still quite a novel concept, teacher educators and teachers are exploring its potential, especially where teachers have difficulty, such as when posing problems.

This current study explores this aspect by analyzing the outcomes of a special course in which former high-tech personnel were undergoing retraining to become advanced-level high school teachers. As part of the course, they explored how different approaches, integrating various technologies, and presenting appropriate problems can enhance student learning. Posing problems in mathematics is an inherently challenging task, and these teachers had not yet practiced it, so we prepared some lessons in which an AI "assistant" (in this case, ChatGPT) would help them pose problems to meet pre-defined educational goals. This took place in accordance with Cai and Rott's (2024) graphical mathematics-problem-solving model.

Literature Review

The Problem in Mathematical Problem Posing

Solving mathematical problems is an important part of mathematics education, meaning that teachers must be adept in posing them according to their pedagogical needs (Brown & Walter, 1983; Mallart et al., 2018; Polya, 1954; Silver, 1994). This may require formulating new problems or re-formulating given (i.e., textbook) ones (Silver & Cai, 1996). Digital technologies can aid the process by providing teachers with the opportunity to explore unique examples, special cases, novel hypotheses, and counterexamples (Gros, 2016; Santos-Trigo & Moreno-Armella, 2016; Segal et al., 2018). Often, altering even one aspect of a given problem's data can lead to an effective revision that can generate new problems (Lavy, 2015; Segal, et al., 2018).

Thus, it is important to give pre-service teachers skills in posing problems, despite the challenges hindering its application (Cai & Jiang, 2017). A survey of problem-solving and problem-posing processes (Liljedahl & Cai, 2021) suggests that there is still much work needed to improve teachers' problem-posing competencies, especially in how they approach mathematical tasks and a case study (Mallart et al., 2018) discussed the difficulties that mathematics teachers have in adapting (textbook) problems to their specific needs, creating problems that can be self-corrected, or designing problems that their students could recognize as relevant to their everyday lives. (This is important because it convinces students that mathematical knowledge has real-world applications, [Nedaei et al., 2021].)

In a literature review on problem posing, Cai and Rott (2024) suggested that in general, problem posing incorporates four phases:

- (1) *orientation* (understanding the problem and strategies for solving it and what improvements are needed);
- (2) *connection* (developing new problems by altering the situation or adapting it to specific educational needs);

(3) *generation* (recognizing the connections made in the previous phase and developing new problems based on these needs); and

(4) *reflection* (reassessing and optimally reformulating the problem).

They suggested that researchers should empirically examine the usefulness of their model to help understand the processes in different contexts. In our current study, ChatGPT was used by the teachers to pose problems and played a role in each of these phases in the process of redesigning the problem.

Generative AI

One "benefit" of the COVID-19 pandemic was that it spurred education systems to, suddenly and completely, introduce remote teaching, forcing teachers the world over to become familiar with technological tools (Klemer et al., 2023; Turnbull et al., 2021). This also accelerated the emergence and accessibility of AI technologies: specifically generative AI (GenAI) which has the potential to significantly change the face of teaching (Baidoo-Anu & Ansah, 2023). GenAI refers to machine learning algorithms designed to allow users to create new content by asking questions (prompts). These apps can provide factual information, answer questions, edit existing texts, or produce new texts and images (Baidoo-Anu & Ansah, 2023; Chan & Hu, 2023; Marienko et al., 2020). For teachers, the app can serve as an "assistant" that can provide suggestions and prompts for preparing courses and designing teaching/learning resources and materials. The use of GenAI in education systems is still in its infancy (Chan, 2023) and there is significant room to increase its integration into teaching and learning practices.

To use GenAI effectively, teachers must have AI literacy, that is, skills to evaluate and communicate with the application. This requires at least a basic understanding of its essence, the ability to interact properly, and critical thinking about the process and the product obtained as a result of that interaction (Druga et al., 2019). To gain the required experience, teachers need opportunities to integrate GenAI technologies into their teaching practices.

GenAI, ChatGPT, and Mathematics

ChatGPT is a form of GenAI that has the potential for interactive learning that includes immediate personalized feedback, adaptive learning, and specific support (Chan & Hu, 2023). Guiding teachers on effectively utilizing GenAI – particularly ChatGPT – while they are also learning complex mathematical content can be a challenge for educators (Jia, 2015). Yet such personalized GenAI feedback can promote pre- and in-service teachers' mathematical knowledge, and a majority of both teachers and educators believe that the "conversational abilities" of ChatGPT could provide instructional support in the case of posing problems (Rane, 2023; Wardat et al., 2023).

GenAI can also help in solving and posing mathematical problems. Regarding problem-solving GenAI has the ability to explain and visualize various geometrical and algebraic concepts and theorems. ChatGPT can carry out interactive problem-solving "dialogues" with the teachers to brainstorm and design step-by-step problems that align with and adapt to their pedagogical needs. According to Segal and Biton (2024), problem-posing with GenAI has the potential to expose learners to new pedagogical ideas, as well as to develop their critical thinking. Experimenting with the ChatGPT platform allows teachers to develop understanding and competency (Kasneji et al., 2023; Rane, 2023; Wardat et al., 2023). It can support collaborative learning with peers (Rane, 2023).

However, teaching and learning mathematics with GenAI also presents challenges. Lack of human interaction is one and teachers have also expressed concern about accuracy, data and intellectual privacy, ethical and liability issues, and the impact on personal development, career prospects, and societal values (Rane, 2023; Wardat et al., 2023). More important, however, is the accuracy and dependability of AI-generated solutions, as they are not always accurate. For example, at present, ChatGPT does not have a deep understanding of various mathematical topics (e.g., geometry) leading to incorrect solutions and misconceptions: in one study, it generalized the Pythagorean theorem to encompass all triangles (not only right triangles) and created "mysterious" arithmetic errors that a simple calculator would not (e.g., providing an incorrect answer to a function when substituting $x = -1$ into $f(x)$) (Baidoo-Anu & Ansah, 2023; Rane, 2023; Wardat et al., 2023). Students and teachers need to be aware of this possibility and evaluate every suggestion it makes. Despite the challenges, ChatGPT does offer many benefits and addressing its accuracy and teachers' concerns effectively can foster the use of GenAI to enhance the teaching and learning experiences (Chan & Hu, 2023).

One way to improve ChatGPT's accuracy and reliability is by giving the platform precise, well-defined input and comparing its feedback to other sources (Wardat et al., 2023). Studies that explore how to safely and constructively integrate GenAI tools into teacher-education programs are called for (Baidoo-Anu & Ansah, 2023).

The Value of TPACK

TPACK is a model widely used for describing the various combinations of technological (TK), pedagogical (PK), and content knowledge (CK) that teachers possess (Koehler & Mishra, 2009, based on Shulman, 1986). The three principle bodies of knowledge intersect at various levels of complexity (Jahangiril et al., 2021; Segal et al., 2021; Tondeur et al., 2020) with the accent on the technological aspect as follows (Guzey & Roehrig, 2012; Segal et al., 2018; Yang et al., 2023):

TK – knowledge about the various technologies available for planning and teaching in the classroom and online environments;

TCK – knowledge about how technology can create different representations for a specific concept (meaning that teachers must be aware of how specific technologies can affect learners' understanding of relevant concepts);

TPK – familiarity with the range of technologies that can affect teaching methods and promote teachers' educational needs.

TPACK – the amalgamation of all three aspects. Teachers with strong overall TPACK use technology to promote inquiry-based activities for their students and use a variety of representations of mathematical and scientific concepts to assist their student's development of content knowledge.

The TPACK model can furnish a useful framework for mapping teacher knowledge, leading to more accurate definitions of what they require to effectively integrate technology into their teaching (Caniglia & Meadows, 2018; Nantshev et al., 2020; Polly & Orrill, 2012). Teachers with expansive TCK and TPK will be better able to use technology in the classroom. Not only can they use it for student assessment, including self-evaluation, peer evaluation, and more (Torres-Madroñero et al., 2020), they can also use it to challenge and improve their students'

thinking and understanding (Özerem, 2012; Stoilescu, 2015) and promote inquiry skills and active learning (Klemer & Rapoport, 2020; Segal et al., 2018). Educators who are aware of their students' (i.e., preservice teachers') TPACK will be better able to enhance their professional development, which will further enhance their TPACK (Segal & Biton, 2024). Furthermore, teachers and educators who are conscious of the TPACK rating may be more motivated and focused to increase it (Jahangiril et al., 2021; Koh et al., 2013; Lavy, 2015; Leikin, 2015; Segal et al., 2018).

One strategy for expanding mathematics teachers' TPACK is by having them design tasks in a digital environment which will demonstrate the technological possibilities for learning and teaching mathematics (Kirikçilar & Yildiz, 2018). In the current study, participants were tasked with utilizing ChatGPT to pose mathematical problems, the aim being not only to improve their problem-posing skills but to demonstrate to them the advantages of using dynamic environments within various contexts.

Research Question

The act of posing mathematical problems can present a challenge for mathematics teachers. This may be aided by the inclusion of GenAI, even though utilizing the power of GenAI in education is still an underdeveloped issue. Nevertheless, the few studies that have explored this suggest that GenAI environments indeed have the potential to serve as a "partner" for helping teachers, not only in their instructional tasks, but also for posing mathematical problems according to their pedagogical needs thereby contributing to their TPACK. Hence, this study aimed to explore how ChatGPT can aid mathematics teachers when posing mathematical problems and contribute to their TPACK, leading to the following research question:

What contribution can ChatGPT make to high school pre-service mathematics teachers' TPACK when it is used to help pose mathematical problems according to specifically defined educational needs? What benefits did the teachers obtain from ChatGPT's responses?

Method

Participants

Fifteen participants took part in the study. All came from the fields of high-tech and engineering and were now in a special academic program to teach mathematics, including advanced, high-level high school mathematics. The participants learned the high-school curriculum's content, pedagogical and didactical methods for teaching the subjects, and various ways of problem solving and posing. A year-long course entitled "From high-tech to teaching" involved integrating technology into teaching and learning mathematics.

Two sessions focused on problem posing utilizing GenAI/ChatGPT, during which the teachers were required to employ ChatGPT during the process of writing a problem-posing task. The process included the four main stages specified by Cai and Rott (2024) (see Figure 1). This research was conducted during these sessions.

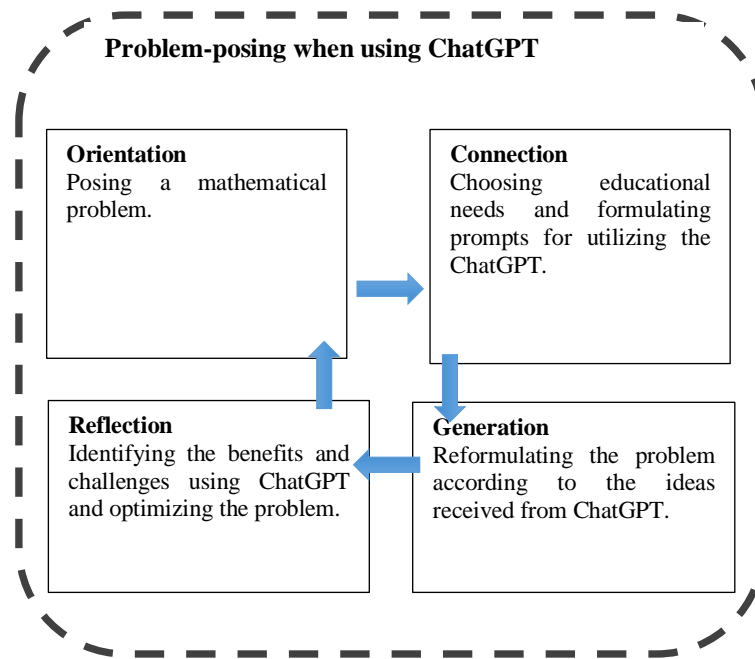


Figure 1. Mathematical Problem Posing Stages Utilizing ChatGPT (based on Cai & Rott, 2024)

Stage 1: Orientation

The teacher poses an advanced-level mathematics problem (sequences, Euclidean geometry, calculus, etc.). They are asked to define the knowledge required to solve the problem and any difficulties a student might have.

Stage 2: Connection

The teacher defines the students' educational needs and formulates prompts to receive recommendations from the ChatGPT for improving the problem. These are based on guidelines and examples received on how to conduct a dialogue with ChatGPT and refine the prompt process according to their specific needs. Here is one example:

I am a pre-service mathematics teacher of As part of a course teaching advanced mathematics, I am tasked with creating a problem in calculus. Here is the problem I posed: "...". I would appreciate it if you could provide me with three recommendations on how to improve my problem.

ChatGPT responds, and the teacher considers the suggestion and, if desired, asks additional prompts to focus on specific aspects or educational goals. Sample questions:

1. Is the problem clear and understandable? How can it be made clearer?
2. Is the problem sufficiently challenging? How can its difficulty be increased?
3. Does the problem contain all the necessary information for its solution? What additional details could enhance its value?
4. Can the problem include real-world applications or connections to other scientific fields?

Stage 3: Generation

The teacher considers all the ideas received from the ChatGPT and decides which suggestions meet their

educational needs and whether they (the teacher) have the knowledge and skills to implement them. They reformulate the problem accordingly.

Stage 4: Reflection

The teacher considers the benefits and challenges of ChatGPT and decides whether to continue for another task. If so, they repeat Stages 1-3, this time posing a mathematical problem that connects the mathematical concept to everyday life. In the current study, we focus on the analysis of Stages 2 and 3, which includes the prompts written by the teachers, the responses from ChatGPT. We also present some samples of the teachers' reflections on how they benefitted (or not) from their interaction with ChatGPT's.

Data Collection

During the sessions, the researchers documented all stages of the problems and the prompts the teachers gave ChatGPT by integrating the links to the AI environment into a Google Form file that guided the teachers, step by step, through the process. Since the teachers' task included two assignments, a total of 30 prompts, 30 ChatGPT responses, and 30 teacher reflections on ChatGPT's response were analyzed.

Data Analysis

Analysis was qualitative. All the responses were divided into “units of meaning” (from one word to a sentence or paragraph that focused on one idea), themes were identified and categorized using “constant comparative analysis” (Charmaz, 2014; Glaser & Strauss, 2017), and then the themes were divided into primary and secondary content areas (Fram, 2013).

Results

Below, we first present a dialogue between one teacher (Nir, pseudonym) to give readers an idea of the dialogue that occurred. This is followed by three tables: the requests made by the teachers to ChatGPT for improving the mathematical problems they formulated (Table 1); the suggestions given by ChatGPT for improving the problems (Table 2); and the feedback from the teachers regarding ChatGPT's contribution (Table 3). All the tables are categorized into themes with illustrative quotes. Explanations of key observations follow to illustrate how interaction with GenAI enhanced the teachers' problem-posing actions and their pedagogical and methodological skills (TPACK) and the quality of the mathematical problems posed.

Example dialogue. Nir's problem-posing process using ChatGPT: Exploring a function

The following example of dialogue demonstrates how the problems were fine-tuned according to their defined educational needs.

Stage 1: Orientation: Posing the problem

Explore the function $f(x) = \frac{2x^2}{x^2+6x+8}$ as follows:

- a. Find the domain.
- b. Find the asymptotic equation.
- c. Find the intersection points with the axes.
- d. Find the extrema.
- e. Find the areas of rise and fall.
- f. Sketch the graph.

Next, Nir defined the required knowledge and the potential difficulties that a student might confront.

Knowledge required to solve the problem: Investigating a quotient function, finding the derivatives of a quotient function, finding the domain, asymptotes, and intersection points with the axes, extrema, areas of rise and fall, and sketching.

Possible difficulties: Students may err when finding the derivative function; students may have difficulty drawing a discontinuous sketch.

Stage 2: Connection

Nir then formulated prompts to stimulate suggestions from ChatGPT:

I am a pre-service teacher teaching mathematics. As part of a course... I was asked to pose a problem for investigating a quotient function.

Here's the problem I posed. ...

Please write down three recommendations on how to improve my problem, specifically:

1. Is the problem challenging enough or does it seem too simple? How can we increase the challenge?
2. Does the problem contain all the information needed to solve it? What missing details can add value?
3. Can real-world applications or connections to other areas of science be included in the problem?

Stage 3: Generation

ChatGPT responded with the following suggestions, and after considering them, Nir reformulated the problem according to his educational needs.

ChatGPT's response

Increasing challenge: Although the problem is good, you can increase the challenge by adding questions that require more in-depth analysis or additional techniques such as asking to describe the behavior of the function next to its vertical asymptotes (i.e., whether the function approaches positive or negative infinity).

Possible difficulties. Consider that the students might have difficulty sketching the area of the graph near the vertical asymptote. You might wish to use a graphic tool to provide a graph that they can compare with their own graph.

Missing details: It may be helpful to include a reminder about the definition of asymptotes and how to find them algebraically, especially for students who may be encountering these concepts for the first time. Make sure you provide clear guidance on how to approach each part of the problem. For example, to find the domain of the function, you might want to include guidelines for identifying values of the function at points where the function is not defined (such as values that make the denominator zero).

Linking the problem to everyday life: Although the problem itself is purely mathematical, you might consider adding real-world context or application to make it more appealing and relevant to students. For example, a scenario in which the function represents a physical state (such as a growth model or financial investment) and then ask students to interpret the results of their calculations in this context. Thus, you can improve the depth, clarity, and applicability of your problem, making it more engaging and valuable for students.

Nir considered the suggestions and decided to implement them as follows

- a. He added a question asking for the behavior of the function graph near the asymptotes. This would help the student deal with the difficulty of drawing the graph of the function around the points of non-definition where it is not continuous.

b. He also introduced a recap of the process of finding asymptotes to assist those who have difficulty or those who are doing so for the first time.

Thus, the additional question was worded as follows:

Find the asymptotes (pay attention to the behavior of the function in close proximity to the horizontal asymptotes). Reminder on how to find asymptotes: vertical asymptotes in a quotient function occur when the function is not defined (that is, the denominator equals 0 and the numerator is NOT 0); horizontal asymptotes in a quotient function occur when the highest power in the expression's numerator is smaller than or equal to the highest power in the expression's denominator.

Stage 4: Reflection

Nir considered his dialogue with ChatGPT and decided which suggestions he should accept.

Nir added a section directing the students to check their sketch using a graphic tool, and then asking them to compare and investigate if there is a difference between their graph and the graph presented and what such a difference might stem from.

Nir chose NOT to implement ChatGPT's recommendation to integrate the problem into daily life by linking the graph to a model of growth or financial investment.

Table 1. Themes and Sample Quotes of Pre-Service Teachers' Prompts for Improving Mathematical Problems

Theme	Theme Description	Quotes
Problem clarity	Requests for recommendations on clarifying the problem statement to ensure it is clear and understandable	"Is the problem clear and understandable? How can it be made clearer?"
Problem challenge	Requests to increase the difficulty level of the problem to make it more challenging	"Is the problem challenging enough or does it seem too simple? How can the challenge be increased?"
Completeness of information	Requests to ensure the problem contains all the necessary information for solving it and to add any missing details if needed	"Does the problem contain all the necessary information for solving it? What details are missing that could add value?"
Real-world	Requests to include real-world	"Are there any real-world applications or

Theme	Theme Description	Quotes
applications	applications or connections to other scientific fields within the problem	connections to other scientific fields that can be included in the problem?"
Identifying potential student difficulties	Requests to identify and focus on difficulties students might encounter while solving the problem	"How can I better focus on the difficulties students might encounter while solving the problem?"
Improving problem formulation	Requests to improve the problem formulation in general to enhance its clarity and precision	"Give me 2-3 recommendations to improve the question so that it is clear and challenging."
Focusing on the required topic	Requests to ensure the problem focuses on the appropriate and correct topics required for the learning content	"The problem must include knowledge of arithmetic series (general term, transition rule, sum of arithmetic series)."

Table 2. Themes and Sample Quotes from ChatGPT for Improving Mathematical Problems

Theme	Theme Description	Quotes
Problem clarity	Suggestions to improve the clarity and understanding of the mathematical problem.	"Although the problem is understandable, it might help to explicitly state the initial amount of money deposited by the parents in the savings plan." "Your problem is mostly clear, but specifying the type of series (arithmetic series) at the beginning could help students less familiar with mathematical terminology."
Increasing difficulty	Suggestions to increase the difficulty and challenge of the mathematical problem.	"The problem can be made more challenging by incorporating additional factors such as inflation or variable interest rates over time." "To increase the challenge, you could modify the problem by introducing a variable for the first term of the series and the common difference."
Adding information and clarifications	Suggestions to add additional information and clarifications to improve the problem.	"Adding context to the problem can stimulate students' imagination and help them understand the relevance of the problem." "Including a hint about the formula for the sum of an arithmetic series could be helpful, especially for students who might not remember it offhand."

Theme	Theme Description	Quotes
Real-world applications	Suggestions to add real-world contexts to illustrate the possible applications of the problem.	<p>"To make the problem more interesting and relevant, you could provide a real-world context or scenario where such a savings plan might be applicable."</p> <p>"Connecting the problem to real-world scenarios not only makes it more relatable for students but also highlights the practical applications of arithmetic series."</p>
Encouraging mathematical inquiry	Suggestions to encourage inquiry-based mathematical thinking and step-by-step problem solving.	<p>"Add a follow-up question that encourages students to explore the solution further. For example:</p> <ul style="list-style-type: none"> • Can you find a general formula for the total number of pages that the boy reads on Thursday? • How many pages will he read on the last day before graduating? • What is the total number of pages he read by the end of the week?" <p>"Instead of simply asking students to find the horizontal asymptote, consider guiding them through the process of understanding why the function approaches a certain value when x approaches positive or negative infinity. You can ask them to analyze the behavior of the numerator and denominator when x becomes large and explain why this leads to the existence of a horizontal asymptote. This approach helps strengthen conceptual understanding and not just computational skills."</p>
Guidance for struggling students	Suggestions to provide instructions and tools for struggling students or those unfamiliar with the topic.	<p>"Consider including a diagram or visual representation of the problem to help students visualize the triangles and their relationships. This will make the problem more appealing and accessible, especially for students who are more visually oriented. A well-marked chart can also provide additional context and clarity to the problem statement."</p> <p>" To make the problem more interesting and aid</p>

Theme	Theme Description	Quotes
		understanding, consider including a visual representation of progress... You can create a table or graph that shows the number of sit-ups she performs each day. This visual aid can help students see the pattern more clearly and make connections between the information given and the idea of the series."

Table 3. Themes and Sample Quotes of Pre-Service Teachers' Feedback on Chat Responses

Themes	Themes Description	Quotes
Problem clarity	Contributions to improving the clarity and understanding of the mathematical problem.	"Based on the chat's recommendation, I separated the question into clear sections." "These recommendations improved the original problem by making it clearer, more challenging, and more relevant to students' experiences and learning." "The chat's recommendation contributed to making it clearer... Clarification on the savings plan: By explicitly mentioning the initial deposit and mentioning the potential interest, the problem is clearer to students. Knowing the starting point and all the other factors involved in the savings plan helps them better understand the problem they need to solve."
Increasing difficulty	Contributions to raising the difficulty and challenge level of the mathematical problem.	"These recommendations improved the original problem by making it more challenging." "The additions suggested by the chat for additional and more challenging sections were unclear and not useful to me." "I accepted the chat's recommendation to increase the challenge of including additional variables or more complex conditions that would require students to use prior knowledge more creatively. For example, asking to find a specific number of additional elements that are not part of the series but meet a certain condition related to it. I accepted the recommendation and added finding a sum of

Themes	Themes Description	Quotes
		<p>the even and odd elements."</p> <p>"The idea of incorporating interest increases the challenge of the problem. Students must now apply their knowledge of exponential growth and compound interest formulas to calculate the final amount accumulated over time. This adds complexity and depth to the problem, making it more challenging for students to solve."</p>
Mathematical precision	Contributions to improving the mathematical accuracy and formulation of the question.	<p>"Based on the chat's recommendation, I changed to mathematical notation."</p> <p>"The chat corrected the errors in English introduced by Google Translate."</p> <p>"The chat's suggestion made me unsure of my question. I think my question was unclear and following the information from the chat, I realized that I needed to make the question more structured and lead students along the path of solving similar triangles. That's why I added data so that it would be clear that it needed to be used and therefore the problem would be more succinct."</p>
Real-world applications	Contributions to explaining the possible real-world applications of the mathematical problem.	<p>"The ideas on how to use arithmetic series in everyday life were nice and helpful for explaining possible future uses."</p> <p>"The chat's recommendation to connect the function to a problem related to growing a plant can be nice and thought-provoking about the behavior of the function close to 0 and approaching infinity. I had hoped that the chat's recommendation would include a function with asymptotes that can be linked to the problem, but this did not happen. Therefore, I would add a discussion of where there is a contradiction between the behavior of the function and the behavior of a real plant (for infinity, we would expect the growth of the plant to stop and wilt.)"</p> <p>"The chat's recommendations contributed to</p>

Themes	Themes Description	Quotes
		improving the original problem by making it more relevant with real-world context. Framing the problem as a family saving for a child's college tuition can show students the practical application of the mathematical concepts they are studying. This not only improves their understanding but also makes the problem more appealing and interesting to solve."
Adding visualization	Contributions to adding visual aids, such as diagrams, to improve the clarity of the problem.	"Based on the chat's recommendation, I added a diagram." "I accepted its suggestion to include diagrams for further clarification of the question."
Encouraging mathematical inquiry	Contributions to encouraging inquiry-based mathematical thinking and step-by-step problem solving.	"The chat suggested challenging the students with questions that build understanding and stimulate step-by-step progress that leads to curiosity and investigative mathematical thinking." "I added subsections to the question that stimulate mathematical thinking in a gradual manner."
Guidance for struggling students	Contributions to providing instructions and tools for struggling students or those new to the topic.	"The chat suggested providing a reminder of the process for finding the asymptote for those struggling or doing it for the first time." "It suggested 'bringing life' to the question by providing a bit more framework for the students' thinking and making it more interesting."

Analyses of the Tables

Table 1 (prompts formulated by the teachers) presents seven key categories by which the quality of the mathematical problems could be improved. Each of these categories addresses important aspects of mathematics education, providing teachers with essential tools to enhance their pedagogical and methodological skills. Specifically, guidance on *problem clarity* shows how to ensure the problems created are understandable and clear. Addressing the *challenge* of the problem helps them understand the importance of an appropriate difficulty level to foster deep thinking. Emphasizing the *completeness* of provided information underscores the need for comprehensive and accurate data for solving the problem, thereby strengthening the teachers' precision and clarity. Including *real-world applications* enriches the curriculum and connects it to students' everyday lives, thereby

increasing their motivation to learn. *Identifying potential student difficulties* allows teachers to anticipate challenges that may arise and to prepare creative solutions. *Improving problem formulation* helps them develop precise and clear mathematical writing skills. Finally, *focusing on required topic* ensures that the problems are aligned with the necessary educational content.

Table 2 (ChatGPT responses) highlights several key contributions to improving mathematical problems. These include suggestions for improving *problem clarity* by adding explicit details and clarifications to help students better understand the question; *increasing the difficulty* of the problems by introducing more complex elements and advanced mathematical concepts; *adding information and clarifications*, such as hints and reminders for important formulas, which are especially beneficial for students who may not remember them; incorporating *real-world applications* to show the practical contexts of mathematics, make the problems more relatable, and demonstrate their relevance to everyday life; *encouraging inquiry-based mathematical thinking* and step-by-step problem solving, which helps build a deeper understanding of the problem in a gradual manner; and, finally, providing *guidance for struggling students*, including detailed explanations and reminders to support those who are new to the topic.

Overall, the chat responses significantly contribute to enhancing the quality of the mathematical problems by providing clear, actionable feedback in these areas, thereby improving the capabilities of the mathematics teachers. An analysis of the teachers' responses (Table 3) highlights the key contributions that they identified from their interaction with ChatGPT. These include *enhancing problem clarity* by breaking down the problem into clear sections and generally improving the formulation to ensure the problem is well understood; raising the challenge and complexity of the problem (i.e., *increasing difficulty*) by suggesting more challenging questions; *improving mathematical precision*, as when the chat helped to more accurately formulate questions and correctly use formulas and mathematical terms; emphasizing *real-world applications*, illustrating the relevance of mathematics to everyday life; adding *visual aids* such as diagrams to improve the clarity and understanding of problems; *encouraging inquiry-based thinking* and step-by-step problem solving by adding subsections that build a gradual understanding of the problem; and, finally, *providing guidance for struggling students*, including reminders and detailed explanations to help those new to the topic. The teachers' comments show that they felt that ChatGPT significantly enhanced the quality of their teaching skills.

Discussion

The introduction of technology into teaching is an inevitable step given the accelerated development of digital platforms that can enrich and improve teaching by providing a deeper understanding of learning and teaching (Schleicher, 2019). This study, which answers Chan's (2023) call, explored how ChatGPT, an innovative GenAI-based technology, was used by mathematics teachers to improve the problems they posed for their high school students. The responses given by the app led them to expand their knowledge and improve their critical thinking. It gave them food for thought about how to modify and enhance the problems to make them more interesting, challenging, and relevant for their students. Posing mathematical problems is a difficult process (Cai & Jiang, 2017), even more so for the participants in this study who were new to teaching. "Partnering" with ChatGPT

helped reduce the challenge.

The responses of the participants (Table 3) indicate that the non-judgmental interactivity ChatGPT provided allowed them to better understand the obstacles that might have been inherent in the problems they posed (see also Baidoo-Anu & Ansah, 2023; Lo, 2023). It also allowed them to explore their own personal teaching methods as there were no restrictions regarding time, number of iterations, or accuracy of their prompts. They could customize their problems as needed. The individual feedback they received from ChatGPT improved their skills in problem posing, similar to the results shown by Wardat et al. (2023) and Rane (2023).

One goal of the study was to assess if utilizing ChatGPT for posing problems contributed to the participants' TPACK. Observations of the participants' interactions and responses showed that their TPACK improved in all stages. For example, in the *orientation* stage, the teachers improved their CK and PCK, as they required in-depth familiarity with the curriculum topics, knowledge to solve the problem, and appreciation of any difficulties that may arise in the process. In the *connection* stage, they had to consider their students' educational needs (PK) and efficiently communicate this to the GenAI. During the *generation* stage, improving the clarity of the wording of the problem, ensuring that no data or information was missing, and receiving sometimes unexpected pedagogical ideas to improve the problem or to connect it to real life experience all promoted their PK, CK and PCK.

In the *reflection* stage, they had to determine whether ChatGPT's proposals met their needs: whether they met their defined educational goals and student skill level (PK), whether increasing the challenge to the students would be feasible (PK), and whether to add interest by applying the problem to daily life (CK). An additional contribution of the process is that the back-and-forth dialogue which led to ever-more specific exchanges between teacher and AI, helped them learn how to conduct a similar fruitful dialogue with their students regarding the problem and its solution (PK).

TPACK is closely associated with technological knowledge, and it seems that the participants also enhanced their TK, TPK and TCK. Firstly, the actions of repeatedly refining prompts until they received the desired feedback or assistance from ChatGPT significantly improved the teachers' TK regarding effective use of the tool. They learned how to use ChatGPT intelligently in the sense of providing effective prompts and thereby ensuring that the problem had clear wording and was fully explained, sufficiently challenging, and that met their specific goals. They had to continuously refine their prompts to receive the best answers (TPK). Another aspect of TPK is ensuring that students have the appropriate knowledge to be able to deal with the problem and understand the relevancy of the problem, both difficulties reported by Mallart et al. (2018) and Nedaei et al. (2021).

An important aspect was the teachers' need to refine their critical thinking (see also Chan & Hu, 2023; Rane, 2023; Segal & Biton, 2024; Wardat et al., 2023). For example, some teachers mentioned inaccuracies and inapplicability in the suggestions made, meaning that each suggestion must be considered carefully. They also noted that the pedagogical ideas were coming from an "unofficial" source, thus conferring upon them responsibility for ensuring suitability. Some teachers, after reading the suggestions, realized they had some "holes" in their knowledge and this can serve as a lever for professional development, or that the suggestion required teaching skills that they did

not have or excessive time or effort to implement. One advantage that was often repeated was how ChatGPT was able to provide suggestions as to how to link the mathematical concepts to real life (investment, measurement, sport, agriculture, and more). This expanded their CK and PK.

AI Literacy

Over the years, the concept of technological literacy, especially AI literacy has expanded to include technological skills, problem-solving, information analysis, and presentation. AI literacy refers to the skills and knowledge necessary to understand AI technology, critically evaluate it, comprehend its capabilities, and use it effectively (Long & Magerko, 2020). It encompasses four key domains: awareness and understanding of skills and techniques, the ability to apply AI tools in everyday contexts, the ability to critically evaluate the outputs produced by AI tools, and consideration of the ethical implications of using the technology (fairness, responsibility, transparency, and safety, see Ng et al., 2021). Research has found a positive correlation between these four domains of AI literacy (Zhao et al., 2023). This study, has shown how posing mathematical problems using ChatGPT encompasses such AI literacy and how it can be harnessed to advance teachers' capabilities.

Conclusion

The introduction of AI-based environments such as GenAI and ChatGPT can enhance the efficiency of teachers when designing tasks for their students, in this case, posing problems to teach advanced-level mathematics. ChatGPT was shown to be a good source of information, and pre-service teachers, even those new to teaching in general and to mathematics teaching in particular, could, with appropriate prompts, exploit ChatGPT to pose appropriate, challenging, and educational problems for their students that meet their curriculum needs.

Introducing the use of ChatGPT into teacher training reinforces Chan's claim (2023) regarding the critical role that teacher educators have in providing teachers with practical learning opportunities in the ChatGPT environment, thereby improving their TPACK. The scope for this is not only limited to posing problems; it can be valuable to all aspects of the challenges faced by all teachers. Given the accelerated development of AI-based environments, researchers in mathematics educations must stay aware of new opportunities for instruction that will be continuously presenting itself and new opportunities for research.

Study Limitations and Suggestions for Future Research

This study was conducted with a limited group (15) of participants. The small sample size bars generalization, so similar studies with a larger group of participants would give more accurate results. Most importantly, the participants in this study were all academics in hi-tech professions who, it may be assumed, would be familiar with technological applications and have a positive approach to their use. Comparing their attitudes to those of teachers who are not so technologically oriented would provide an interesting approach to the use of ChatGPT. Finally, the use of ChatGPT or similar GenAI environments should be explored for other pedagogical needs, such as developing teaching processes, introducing such technologies in the classroom for student use, and more.

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