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"*True to Myself*": Transforming Secondary Mathematics Teachers' Beliefs and Practices

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Abstract

Given the minimal impact in the classroom from the implementation of reform movements internationally, the question remains how researchers and teacher educators can address the learning needs of secondary mathematics teachers. The goal was to implement research that was focused on providing an empowering space for the advancement of teachers' beliefs and practices. This study showed that it was possible to impact the beliefs and practices of a group of teachers who had varying levels of loyalty to more traditional practices. This was accomplished by considering practitioner-derived knowledge to be trustworthy and relevant. By employing a critical and participatory research methodology informed by Kemmis and McTaggart, and by designing a study that was framed by Fay's three stages of empowerment, I was able to provide a space where teachers became aware of their beliefs and practices, negotiated barriers, became more grounded in their personal philosophical perspectives, took action to implement novel pedagogies, and knew finally "who they are" as professionals.

Key words: Mathematics education, Secondary school teachers, Beliefs, Educational practices, Professional development.

Introduction

Mathematics education reform, as influenced by the TIMSS studies (The Trends in International Mathematics and Science Study) has been underway internationally for at least 50 years (International Association for the Evaluation of Educational Achievement, 2013). Governments and learning institutions around the world introduced new curricula with the implicit assumption that these initiatives would improve teacher practice (Brown, Hanley, Darby, & Calder, 2007). However, though mathematics education reform movements in New Zealand, Australia, Europe, the UK, and North America have been underway for a number of years (e.g., Australian Education Council, 1990; Common Core State Standards Initiative (CCSSI), 2010; Ministry of Education of New Zealand, 1992; Ministry of Education of the United Kingdom, 1959; National Council of Teachers of Mathematics, 2000, 2006; Office for Standards in Education (OFSTED), 1994), studies continue to show that there is little evidence of adoption by teachers or of changes in classroom practices (Bishop & Glynn, 2003). Transmissive practices still predominate (Brown et al., 2007; Hiebert et al., 2003; Mullis, Martin, Foy, & Arora, 2012; OECD, 2009; Reio Jr., 2005).

There are many documented barriers to the implementation of today's reform movements, and many agree that it is time to focus on the required shifts in beliefs and practices that are assumed by these movements (Brown et al., 2007; Pehkonen & Törner, 1999). There is promising evidence that constructivism and other empowering pedagogical perspectives or paradigms such as ethnomathematics and criticalmathematics can have an impact on teacher practice (Boaler, 2008; D'Ambrosio, 2001; Fosnot, 1996; Hiebert et al., 1997; Skovsmose, 2004). This study provides a theoretical and practical framework for an actionable methodology that provides support for secondary mathematics teachers as they navigate the implementation landscape.

Secondary mathematics teachers often perceive tensions between their personal philosophies and other views or expectations they face in their work. These expectations can include the pressures of new curriculum reforms that may not match the teacher's view of teaching mathematics. These tensions and the few pedagogical choices

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offered for secondary teachers provided a rationale for this study. I wished to offer learning opportunities and choices that would provide the space for the empowerment of teachers through practices that promote teachers' adoption of a critically reflective perspective. There are few such studies developed for secondary mathematics teachers, and even fewer that would implement a participatory and critically reflective approach (Moon, 1999; Wildman & Niles, 1987). Many agree that it is time that teachers are acknowledged (Appelbaum & Davila, 2007; Hanley, Darby, Calder, & Brown, 2005; Reio Jr., 2005), and that professional learning be viewed from the perspective of providing an empowering space for teachers rather than as imposed reform (Brown et al., 2007; Cooney, 2001; Woods, 1985).

Rather than a focus on purely evaluative evidence, practitioner-derived knowledge must be considered trustworthy and relevant in research (Lerman, 1994; Smyth, 1989b). This study intended to value practitioner-derived knowledge. It focused on the teachers' work through their words and experiences. Informed by Kemmis and McTaggart (2005) seven key features of action research, I was intent on designing a study that could lead teachers through Fay's (1987) three stages of understanding, empowerment, and transformation. If the teachers' discourse could be shown to progress through these three stages, then the research design could be said to bridge the dichotomy between research and practice as Kemmis and McTaggart had envisaged. I sought to "illuminate particular practices and practical settings" (p. 568) to develop insights about how teacher practice might be transformed. By examining their beliefs, and engaging with multiple philosophies and pedagogies for teaching mathematics, teachers could describe their experiences and growth through the reflective practices supported by Kemmis and McTaggart's (2005) key research tenets.

Teacher education "should provide practitioners with the tools and resources they need to recognize, analyze, and address the contradictions, and in so doing, open-up the possibility that conditions in schools can be different" (Smyth, 1989b, p. 4). There is still a need to apply research methods that emphasize "the importance of being able to see, talk about, and then try a practice with support" (Levine, 2010, p. 121). My challenge was to develop effective processes and tools for teacher educators and researchers to support mathematics teachers in the examination of their beliefs and in the exploration of alternate mathematics pedagogies for the purpose of developing a greater self-awareness of their beliefs and practices. To address these concerns, I asked the following research question: Do Fay's stages of empowerment provide a useful framework for describing the progress of a group of teachers engaged in processes designed to reveal self-knowledge of participants' beliefs and alternate practices? According to Fay (1987), consciousness can be developed through a three-stage process that is enlightening, empowering and emancipatory. This $E_1E_2E_3$ (Enlightenment-Empowerment-Emancipation) model was discussed by Johns (2004) to explain the reflective process that occurs during the practice of nursing. Johns described Fay's three stages as shown in the table below. The parallels between Fay's work, Johns' interpretation and this study are shown in Table 1.

Table 1. Three Stages of Empowerment (Johns, 2004) – Based on Fay (1987)

Stages (Fay, 1987)	Johns (2004)	This Study: Based on Fay (1987)
Enlightenment (understanding)	Understanding why things have come to be as they are in terms of frustrating self's realization of desirable practice.	Participants come to understand their personal knowledge, beliefs and practices and make sense of any tensions between them.
Empowerment	Creating the necessary conditions within self whereby action to realize desirable practice can be undertaken.	Participants articulate and learn to question barriers and discourses that have become embodied (reified). They begin to see themselves within a larger framework, they notice their positioning within that structure, and that things *can* change.
Emancipation (transformation)	A stable shift in practice congruent with the realization of desirable practice.	Participants are driven to "socially transformative action". They also experience a *liberation* - or a "state of reflective clarity". They step out of their comfort zone and become agents of change.

Given my purpose of supporting teachers' reflective process during their practice, I selected the $E_1E_2E_3$ framework as an important supporting element for the design and analysis of my work with teachers.

Literature Review

This study took an emancipatory perspective and supported its purpose by providing choice to the participants. It was informed by Kemmis and McTaggart's (2005) seven key features of participatory action research, and had the goal of providing an empowering space for its participants (Guba & Lincoln, 2005). Research studies tend to investigate teachers' pedagogical choices as limited to either a *transmissive* or *reform* mode (Lerman, 2003). I intended to offer a choice between these teaching perspectives as well as other possibilities and pedagogical stances from the literature which are not typically promoted in reform documents but are, nonetheless, appropriate for addressing critical thinking and social justice issues for 21st century learners (e.g., D'Ambrosio, 2001; Skovsmose, 1994; Stinson, Bidwell, Powell, & Thurman, 2008). I intended that participants would be introduced in more detail to the two dominant perspectives, one based on a behaviorist model of learning (referred to as transmissive according to Lerman, 2003) and one based on a constructivist view of learning (referred to as reform according to Lerman, 2003), but also that they would have the opportunity to become familiar with two other emergent critical pedagogies for cultural and political empowerment of students (e.g., ethnomathematics and criticalmathematics).

Importance of Teachers Reflecting on their Beliefs

This study investigated teachers' espoused beliefs within the context of teaching and learning mathematics. As Thom (1972) reminded us, "whether one wishes it or not, all mathematical pedagogy, even if scarcely coherent, rests on a philosophy of mathematics" (p. 204). Secondary mathematics teachers' voices and beliefs are under-represented in the literature, and the concern that those *expert* teachers "may be particularly susceptible to an expert *blindspot*" (Nathan & Koedinger, 2000, p. 209) needs to be addressed and researched. Furthermore, the theoretical perspective that teachers hold, whether implicitly or explicitly, is essentially linked to their practice. As Kemmis (2005) articulated, "[p]ractice is always '***theoretical***' – it refers to the theory that informs it (of which practitioners may or may not be aware)" (p. 8). This work attended to secondary mathematics teachers' beliefs, and was designed to recognize their voices as they worked through a process of reflection and self-empowerment.

Teachers' philosophical beliefs are a more powerful determinant of their practice than reform documents or prescribed pedagogical texts (Hofer, 2006; Schwab, 1974). Because unexplored teachers' beliefs can be a barrier to pedagogical growth, teachers' beliefs must be examined. Teacher development researchers have recommended that reform be viewed from a teacher's standpoint and as a *liberation* rather than a prescription (Brown et al., 2007; Cooney, 2001; Lerman, 1994; Reio Jr., 2005; Smyth & Shacklock, 1998; Woods, 1985). In order to examine secondary mathematics teachers' beliefs from the point of view of the participants, a reflective approach to research and practice is required. Through the reflective practices designed for this study, I anticipated that teachers would become able to make sense of their beliefs and practices (Schein, 1996).

The following quote is from one of the participant's first journal reflections:

> I think the [traditional] mode of teaching math is more effective in school, especially at the younger grades. Following and carrying out procedures, repetition, strong work habits, and learning from your mistakes are all important factors in the work world. I think the more traditional approach to learning math is far more effective at achieving all these goals. It's fine to try the [reform/problem-based] mode once in a while, but to push to have this be the main instructional approach in a math classroom is just silly (Raymond's First Journal).

When he wrote this at the beginning of this study, this teacher may have been unaware that he held a particular philosophy about mathematics, teaching, and learning. He saw his statements as *facts*, and had not necessarily reflected critically upon them. Kagan (1992) noted that "[t]eacher belief is defined broadly as tacit, often unconsciously held assumptions about students, classrooms, and the academic material to be taught" (p. 65), and that "[i]n a landscape without bearings, teachers create and internalize their own maps" (p. 65). Many educators appear to be adrift in a *landscape without bearings*. This study intended to expose these bearings so that teachers could situate their beliefs and practices and learn to know themselves better (Kegan & Lahey, 2007). What we think is a *truth* can simply be a strongly held belief that we have never challenged. When considering the difference between knowledge and beliefs, Wilson and Cooney (2002) described the concepts of knowledge and beliefs as being blurred; if an individual feels that his beliefs have reasonable evidence to support them, he will consider them to be knowledge.

Reflective practice allows participants to recognize their beliefs, and learn to be aware of how those might impact their practice. Swan (2006) argued that beliefs:

> ...underlie dispositions to engage in certain practices and not others. They help people to understand themselves, to understand their environment and to form social groupings around shared values, thus reducing discord. Beliefs thus become comfortable, reified as "the way things are", and are often resistant to change. (p. 59)

Some assert an urgency that teachers should become more aware and critical about their beliefs and about the structures that surround them in their work. For Lesser and Blake (2006), for example, if teachers do not reflect on their personal philosophies, they will continue to re-create the "reified typifications of mathematics" (p. 167) in the classroom. There are several implications for teacher educators when taking into account the considerable importance of teachers' beliefs. Ernest (1988) explained that a "shift to a problem solving approach to teaching requires deeper changes. It depends fundamentally on the teacher's system of beliefs, and in particular, on the teacher's conception of the nature of mathematics and mental models of teaching and learning mathematics" (p. 1). Many agree that teachers must attend to their beliefs through professional learning experiences (Barkatsas & Malone, 2005; Kagan, 1992; Pajares, 1992), and Swan (2006) argued that changes in beliefs might be the best indicator of growth for teachers. Hofer (2006) agreed that "[r]esearchers in this area need to do more to educate teachers about the power of epistemic beliefs as well as to help educators at all levels consider the value of addressing epistemic understanding" (p. 93).

Determining Whose Reality Counts

This research paradigm is interpretivist. It assumes that reality, as we know it, is constructed intersubjectively through the meanings and understandings that we develop socially and experientially. "Knowledge is a dynamic product of the interactive work of the mind made manifest in social practices and institutions" (Paul, 2005, p. 46). It includes an "*enacted*, or *constructed*, reality, composed of the interpretive, meaning-making, sense-ascribing…activities which produce meaningfulness and order in human life" (Lincoln, 2005, p. 61). Since we cannot separate ourselves from what we know, *truth* cannot be grounded in an objective reality. The axiological implication is that researchers must make "extraordinary efforts to reveal, or uncover, beliefs and values that guide and generate individual and group constructions" (Paul, 2005, p. 36).

Since it is argued that beliefs about practice must be attended to in research studies that concentrate on teacher development, qualitative researchers are tasked with designing and enacting studies with trustworthiness and catalytic validity. According to Guba and Lincoln (2005) *catalytic validity* describes a research process that is able to prompt action on the part of research participants. Kemmis (2006) argued that transforming realities "requires truth-telling both with respect to the truths that arise from our work (our findings) and the methods by which we arrive at them. It requires that we critically evaluate how we have done our work – whether our findings are justified by our methods" (pp. 474-475). Too often, it is assumed that another's (researcher's) observed interpretation of classroom practice is more objective, and therefore more valid than a participating teacher's interpretation. Taking into account the interpretivist research stance, rather than a focus on *observed* evidence, this study considers practitioner-derived knowledge to be trustworthy and relevant (Lerman, 1994; Smyth, 1989b).

Teachers' self-reported practice, particularly as reported to colleagues in the *public sphere* about one's practice should be considered valid. In research methodology, this implies that particular texts, including what might be considered *indirect* observations such as reflective journals should count as a good representation of what those participating teachers *know*. Focus group methodology is strongly supported. Kemmis (2006) argued that it is important to establish "public sphere in which people realise and enact their communicative freedom", and that we must "open communicative spaces in which 'the way things are' is open to question and exploration" (p. 474). If this can be accomplished in a research setting, then surely, the teacher's own *public* declarations about her practice can count as the *truth*.

Exploring Barriers and Power Relations

It was my goal to provide a reflective space for teachers to explore and understand their beliefs as well as the "interpretations of [their] social institutions and traditions" (Stinson et al., 2008, p. 619). It is argued that people "are bound by social norms (tradition), by power relations with others (authority), and by previous learning that has become embodied (embodiment)" (Johns, 2004, p. 8). This provides the rationale for examining barriers that "blind and bind ...[and that] limit the practitioner's ability to respond differently to practice situations even when they know there is a better way of responding to situations in tune with desirable practice" (p. 8).

An awareness of barriers was necessary in the process of this study because they tend to block teacher professional growth if they are not discussed. As Kemmis and McTaggart (2005) described: "[p]eople not only are hemmed in by material institutional conditions, they frequently are trapped in institutional discourses that channel, deter, or muffle critique"(p. 571). Given this perspective, I chose to ask questions of my participants with the purpose of eliciting conversations about the barriers they felt they encountered in their practice. I expected these probing questions to result in discourses where participants claimed certain *truths* about the world of teaching. As Miller (1994) explained: "[i]nstitutional discourses are made up of the assumptions, concerns, and vocabularies of members of socially organized settings, and the ways in which they interact" (p. 280). These are the stories and assumptions about schooling that are told and retold and are accepted as the norm in schools. The conversations that repeat themselves in teachers' discourses reflect their *truth* about the world of teaching.

What is interesting about institutional discourses is that they are repeated and rarely challenged. They create and sustain unconscious assumptions that nothing can be done about these *facts*, because the stories just describe *the way it is* (Singh, 1993). Generally, teachers will not be aware of the assumptions they make until they are challenged. This limits them because they cannot be agents of change unless they understand that *things can change*. It is possible to help teachers learn to recognize the assumptions they are making, and research and teacher development can be designed to address them. As Kegan and Lahey (2007) described:

> [p]eople often form big assumptions early in life and then seldom, if ever, examine them. They're woven into the very fabric of our lives. But only by bringing them into the light can people finally challenge their deepest beliefs and recognize why they're engaging in seemingly contradictory behavior (p. 50).

Kegan and Lahey call these narratives that impede our growth and learning *subconscious competing commitments*, and describe how it is important to design professional environments where those barriers can be made explicit, and to provide the space where participants can engage in challenging those subconscious competing beliefs or discourses. A critical and participatory methodology can be applied in such a way that participants are provided with opportunities for empowerment through greater self-knowledge and self-development.

Fay (1987) viewed critically reflective practice as moving through three stages: enlightenment (E_1), empowerment (E_2) and emancipation (E_3). These stages of enlightenment are necessary for teachers so that they may come to understand their personal knowledge, beliefs and attitudes, and make sense of any tensions between them. Fay described how reflective practitioners would experience a *liberation* or a "state of reflective clarity" (p. 205) bringing harmony to those tensions. Fay's model requires the articulation of barriers such as social norms, power relationships and previous learning. He argued that the reified barriers can become so embodied that it does not seem possible to question their existence, let alone work to transform them. These norms are responsible for the maintenance of the status quo in societies and organizations, and they tend to remain unquestioned. These norms are re-created through discourses or narratives that *reconfirm* their truth. Through critical reflection and understanding, individuals can learn to question and possibly alter what they might have originally believed to be unalterable realities. Fay also suggested that an atmosphere be created in which research participants can exercise their voice and develop a sense of human agency. Critical reflection is not passive. To be an *agent of change* is to practice what one believes. As was described in Ford and Profetto-McGrath (1994), "action ensues from critical reflection; nevertheless, the action involves risk taking, as the individual engages in a process that challenges and changes the status quo" (p. 343). This unifies critical thought and practice, and forms the notion of praxis. Fay's (1987) $E_1E_2E_3$ stages are central in the supporting framework of this study, and provide the rationale for the process, method and results.

Methodology

The Perspective and Practices of the Critical Researcher as Facilitator

After comparing a number of action research studies, McCutcheon and Jung (1990) found that, although all of the studies they reviewed referred to their work as participatory action research, there were clear philosophical differences between them. These differences influenced the research questions and how the research was conducted. During action research, the facilitator's research paradigm strongly impacts the relationships between the participants and the researcher, and the unfolding of the research itself. Researchers who practice action research, therefore, should make clear their epistemological beliefs and assumptions (Lerman, 2001).

I employed a critical and participatory (Guba & Lincoln, 2005) stance, and informed this work by employing Kemmis and McTaggart's (2005) seven key features of action research. The ontological perspective was understood to be that knowledge is co-created and participative, and was shaped by social, political, and cultural values. It was appropriate that the methodology was collaborative and dialogic. The internal validity in this study resulted from its fidelity to a critical perspective supported by its theoretical design, the logic of the arguments made in the research literature, and the use of individual and collective data. This study moved its participants to take action, which gives evidence of catalytic validity. "Catalytic validity points to the degree to which research moves those it studies to understand the world and the way it is shaped in order for them to transform it" (attributed to Patti Lather in Kincheloe & McLaren, 2005, p. 324).

A critical researcher who is employing Guba and Lincoln's (2005) idea of *practical philosophy*, and working to help teachers develop McCutcheon and Jung's (1990) concept of a *lived theory* would need to practice Gadamer's (2004) and Towers'(2010) notion of *phronetic* knowledge. *Phronesis* is a kind of practical knowledge that "presupposes a moral attitude, which it continues to develop" (Gadamer, 2004, p. 20). As Towers (2010) explains, a "sense of ethical purpose is, then, central to the work of teaching (and teacher education)" (p. 246). From this perspective, the "participant-researcher" (Kemmis & McTaggart, 2005, p. 572) works in collaboration with teachers as a "critical friend" (p. 571). Critical action researchers must practice *in-action* in accordance with their values and theoretical perspective.

I kept my role in balance, and critically, intentionally, and regularly reflected on my practices and my influence upon the group. I applied a critical researcher's perspective upon my own actions and honed my facilitation skills during the course of this research. This allowed me to practice what I understood to be an ethical and phronetic praxis. I kept a researcher's journal, I regularly reflected critically upon the scripts as I prepared for our next work together, I attended to the group's discussion *in-action*, and asked probing questions during our discourse to augment awareness and metacognition as well as to clarify my role, and I worked hard to include all voices during the course of the study.

Design of the Study – Seven Key Features

In Kemmis and McTaggart's (2005) re-examination of their work on participatory action research, the authors noted that, even though many people identify the image of the spiral of cycles of self-reflection (planning, acting, observing, and reflecting, etc.) to their work, there are seven other key features of action research that are "at least as important as the self-reflective spiral" (p. 566). This study was designed to attend to these tenets.

Table 2. Seven Key Features (Kemmis & McTaggart, 2005)

Feature of Action Research	Meaning (pages cited from Kemmis & McTaggart, 2005)	In this study
1: It is a social process.	Participants, individually and collectively try to understand how they were "formed and re-formed" (p. 567) as individuals and in relation to each other.	This study uses both collective focus groups data and individual reflective journaling data.
2: It is participatory.	The work engages the participants in examining their knowledge, and they way in which they interpret themselves in their world.	In this study, the participants regularly examine their beliefs and their practices.
3: It is practical and collaborative.	The work engages participants through social interaction.	Through focus groups, reflective journals, and lesson documents, the participants work together and reflect critically and thoughtfully on the professional interactions that took place during the study.

Feature of Action Research	Meaning (pages cited from Kemmis & McTaggart, 2005)	In this study
4: It is emancipatory.	The work aims to help people to release themselves from the "social structures that limit their self-development and self-determination" (p. 567).	This is evident in the intentional framing of the study using Fay's $E_1E_2E_3$ model.
5: It is critical.	The work aims to help people to release themselves from their language discourses, their "modes of work" (p. 567), and their social relationships of power.	Focus group and reflective journals emphasize questions about barriers and power relations. These questions are exercised throughout the study.
6: It is reflexive.	The participants in the study engage in a deliberate process intended to transform their practices through self-critical action and reflection.	The application of the $E_1E_2E_3$ model and the regular reflective journaling intend to support participants through a critical and participatory research process.
7: It aims to transform both research and practice.	The work intends to bridge research and practice by helping to "illuminate particular practices and practical settings as a basis for developing critical insights and ideas about how things might be transformed" (p. 568).	This study offers a number of pedagogical practices with the aim of establishing promising research practices for teachers to transform their practice.

Participants and Context

In qualitative studies, research validity is often associated with the development of ethical relationships between participants and the researcher. The reason for this is that more credible data can be collected if the study has been designed to establish a trusting climate. Guba and Lincoln (2005) deepened this idea by asserting that "[t]he way in which we know is most assuredly tied up with both what we know and our relationships with our research participants" (p. 209). I came to be involved in this study through my work with teachers locally, nationally and internationally. For more than two decades I had been supporting teachers with conferences, workshops and websites, and my reputation was of someone who was a strong curriculum leader and coach, and could be trusted. According to Elsbach (2004), the teachers almost certainly had an "enhanced perception of trustworthiness" (p. 282) because I am a teacher (I am *like* them), and because I had a reputation for trustworthiness - competence, generosity, and integrity.

This research involved a group of seven secondary mathematics teachers (pseudonyms used) comprising a mathematics department in a small high school with grades from 8 to 12 that enrolled approximately 700 students. There were three experienced male teachers, two female teachers who were newer to the profession, and two teacher candidates on practicum. The school was in a rural area outside a larger city center with a greater population of approximately 55 000 (which included the rural area).

This school district was relatively small and had limited resources to supply professional development on an ongoing basis. It had, however, a district-level administrator who very much supported the work of professional learning communities when possible. After my call for participants, this school's mathematics department was eager to engage in professional learning that was ongoing, and in particular, in the proposed study.

This work occurred between February and June of the school year. We met on Tuesdays after the school day, every two or three weeks, according to the teachers' preferred days. I had met some of the teachers at previous times during state-level professional activities, but I had not met all teachers, and was not previously associated with the school or the school district. The teachers were not compensated.

Due to the time of day, a light dinner was provided. Because of limited financial resources, this school also employed teachers on a temporary basis who might not be re-hired at the same location the following year. Even though some teachers were temporary and some permanent, this mathematics department worked together in a very friendly manner with each other, and, was eager as a group to commit to this study.

The table below describes the group of teachers in this study.

Table 3. Participant Demographics

Participant (in decreasing order with respect to age)	Role	Number of Years of Teaching Experience	Grades the Teacher Currently Teaches	Male or Female	Post-Secondary Training	Education Degree Certification
Alex	Head of the department	25-29 yrs.	10-12	Male	Mathematics Degree	Secondary Education with Mathematics
Dale	Established in the school	10-14 yrs.	8-9	Male	Education or Physical Education	Elementary Education
Grant	Mature teacher candidate (Alex's)	Practicum	9-11	Male	Science or Engineering Degree	Secondary Education with Mathematics
Raymond	Newer to the school	15-19 yrs.	9-11	Male	Education or Physical Education	Secondary Education with Mathematics
Lecia	Relatively new to the school	5-9 yrs.	8-10	Female	Science or Engineering Degree	Elementary Education
Cara	New to the school	0-4 yrs.	8-10	Female	Science or Engineering Degree	Secondary Education with Mathematics
Julian	Teacher candidate (Lecia's)	Practicum	8-10	Male	Mathematics Degree	Secondary Education with Mathematics

Method and Design of the Inquiry

The literature review supported the design, and informed the process through which teachers participated in the study. The study (designed with five sessions) was facilitated over a period of approximately six months. Data were collected through audio-recordings of focus group sessions, reflective journals, researcher's field notes and reflections, and document analysis of the teachers' written lesson plans and reported lessons. In this study, Focus Groups (FG) are defined using Kitzinger's (1994) definition. As a group, we explored questions and ideas collectively, and the group's interaction was expressly included as research data.

Session I

In Session I of the study (see Table 4), participants learned about the purpose of the study and were given an overview of the five sessions. At that time, I described the nature of critical and participatory research and explained my research perspective. I clarified that my goal was to provide an empowering environment for them by exploring and identifying their beliefs and practices, to keep in mind Kemmis and McTaggart's seven key research features, to keep it a social process, to engage the participants in examining their own knowledge, to maintain social interaction, to help release them from the structures that limit them and from their own limitations, and to deliberately engage in self-critical action and reflection.

Table 4. Overview: Sessions I - V

Session	Description of What Each Session Entailed
Session I)	(FG1) Orientation to the Study (Research Perspective); Focus Group Questions; Investigate Beliefs PD*: (Introduction to Alternate Mathematics Pedagogies) Readings: Constructivism and "Radical Mathematics"
Session II)	(FG2) Focus Group; Explore and Investigating Beliefs as a Group - Four Mathematics Pedagogical Perspectives (J1) First Electronic Reflective Journaling Exercise; Teachers are asked to justify their beliefs and practices Reading: Ethnomathematics
Session III)	(FG3) PD*: (Mathematics Lesson Example: Focus on Constructivism); Lesson and Focus Group Questions Action Research by Teachers: • Plan: Teachers plan • Act and Observe: Teachers deliver a lesson and elicit feedback from their students • (J2) Reflect: Second Electronic Reflective Journaling Exercise
Session IV)	(FG4) Reflect: Group Sharing - Individual Teachers' Lesson Presentations (J3) Third Electronic Reflective Journaling Exercise
Session V)	(FG5) PD*: (Mathematics Lesson Example: Focus using Social Justice Perspectives – Ethnomathematics and Criticalmathematics) (J4) Fifth and Final Electronic Reflective Journaling Exercise

*PD indicates Professional Development Support

After the short introduction, I asked the participants (in a focus group situation) a number of questions about who held the power in their work situations, and about their perceptions of the barriers and factors that facilitated their work. The focus group questions (FG1) were:

1. In my job, who (or what) has power or influence over my work?
2. In my job, over whom (or what) do I have power or influence?
3. What are the biggest barriers that stop me from implementing the best lessons?
4. What are the factors that facilitate my best work as a teacher?

One of Zeichner's (1996) critiques of reflective practice is that practitioners too often disregard the social and institutional contexts of teachers' work. In this research, critical thinking skills were supported by asking "difficult questions" (Jaworski, 1998, p. 3) that might challenge the status quo. Jaworski's work reflects Foucault's (1972) description of how change can occur. In his analysis, Foucault argued that it was important to seek to elicit shifts and *rupture* points in one's experiences that would cause "displacements and transformations

of concepts" (p. 4) and would affect practices. When hard questions are asked, these can serve as *jolts* or *nudges* for the teachers participating in reflective practice. Researchers and educators of adults must develop and apply practices that help participants "become critically aware of the cultural and psychological assumptions that have influenced the way we see ourselves and our relationships and the way we pattern our lives" (Mezirow, 1978, p. 101).

There are few practical examples in the literature that give detailed information about exactly which questions facilitators might ask when working with teachers in this way. Smyth (1989a, 1989b) suggests that the following questions are appropriate and necessary to nudge teachers:

- What do my practices say about my assumptions, values, and beliefs about teaching?
- Where did these ideas come from?
- What social practices are expressed in these ideas?
- What is it that causes me to maintain my theories?
- What views of power do they embody?
- Whose interests seem to be served by my practices?
- What is it that acts to constrain my views of what is possible in teaching? (p. 7)

Smyth's *hard* questions offered important guidance to the structure and facilitation of this study. Following the focus group discussions of questions (FG1), a *Framework of Perspectives* (see Table 5) was introduced as a professional development activity and used to guide an audio-recorded group discussion (FG1) about alternate pedagogies. As a structuring device to implement Fay's $E_1E_2E_3$ model, I chose Lerman's framework of perspectives to allow me to scaffold and organize the range of beliefs that might be held by secondary mathematics teachers.

Table 5. Framework of Perspectives (FoP) – Teachers' Pedagogical Modes (Lerman, 2003)

Perspective/Mode	Pedagogical Style	Supporting Theory
Traditional *(Performance Model)*	e.g. transmissive style	Behaviorist
Liberal/Progressive *(Competence Model I)*	e.g. reform mathematics	Constructivist
Populist *(Competence Model II)*	e.g. ethnomathematics	Sociology/ Structuralism
Emancipatory/Radical *(Competence Model III)*	e.g. criticalmathematics	Critical Theory

The intent of introducing this framework to teachers was also to advance the participants' knowledge about various perspectives or paradigms for teaching mathematics. Later, during Session II, the charts in Appendix A (also provided in Figure 1) would provide more detailed information for the teachers about the pedagogical stances described in the Framework of Perspectives. The teachers' understanding of the perspectives was also deepened with the readings that were assigned and with the discussions that ensued.

The idea that mathematics teachers should be offered the opportunities to explore their beliefs, and that they should not be coerced into a predetermined conception or definition of improvement is supported by Brown et al. (2007):

> [M]athematics education research and development should seek to recognise differences in teachers' understandings, experiences and context of action and assist them in making informed professional judgements about how their practice might be developed in situ, rather than supposing that external evaluative judgements should be based on movement to a consensually preferred conception of teaching. (p. 198)

The study's participants had the opportunity to explore their beliefs and their philosophical stances within this framework of four choices of pedagogies, and they were involved in experiences where they could reflect both as a group and individually on their beliefs and practices.

Besides providing a framework for teachers to learn about different perspectives and practices, the offering of all four possibilities gave teachers choices. This study did not assume that the reform movement in education was the best or *only possible stance* for secondary mathematics teachers to adopt. Because a critical research approach was taken, this "demands an ideology of teacher education different from that traditionally employed, which usually involves models of 'best practice', emphasis on competencies, and unrecognised conflicts between institutional ideals and workplace socialisation" (Hatton & Smith, 1995, p. 38). Moreover, it is well recognized in adult education that adult learners respond better when they are given choices rather than when they are told there are specific best practices (Brookfield, 1988; Merriam, 2004).

Session II

As McCutcheon and Jung (1990) described, it is against a *lived theory* (or practical philosophy) that teachers view research or other educational problems. Teachers hold unexplored beliefs, and their lived theories may or may not be articulated nor apparent to them. Teachers, even if aware of their views, may have difficulty expressing those beliefs unless there is an initial stimulus for discussion (Cooney, 1985; McCutcheon & Jung, 1990; Thompson, 1984). Kagan (1992) recommended that facilitators use a reflection tool, or some form of information, as a foundation for the exploration of beliefs and practices. This is important for the enlightenment (E_1) stage of empowering practices.

In Session II, a second exploratory tool was introduced. Practices and beliefs statements were sorted and organized in charts, and were categorized as either supported by a behaviorist view of learning, a constructivist theory of learning, or by one of the *Social Justice* teaching paradigms (e.g., ethnomathematics or criticalmathematics). Figure 1 shows a sample of the charts used as an investigation and discussion tool during the second session. The entire set of charts used for this purpose can be found in Appendix A.

Component 1: My View of Teaching	
Competence Mode:	**Performance Mode:**
My Goal: ☐ My goal in class is to help students construct mathematical knowledge and to empower them as learners and thinkers.	***My Goal:*** ☐ My goal in class is to help students practice and master important mathematical procedures and skills.
My Beliefs: ☐ Teachers should encourage students to find their own solutions to problems even if they are inefficient. ☐ Teachers should allow students who are having difficulty solving a problem to continue to try to find a solution. ☐ Teachers should facilitate students' inventions of ways to solve problems.	***My Beliefs:*** ☐ Instructors should focus on facts instead of theories. ☐ The teacher should demonstrate how to solve simple word problems before students are allowed to solve problems. ☐ An effective teacher demonstrates the right way to do a problem. ☐ Teachers should tell students who are having difficulty solving a problem how to solve it.
My Practices: ☐ I find out which parts students already know and don't teach those parts. ☐ I teach students differently according to their individual needs.	***My Practices:*** ☐ I teach the whole class at once. ☐ I tell students which questions to tackle. ☐ I go through one method for doing each question. ☐ I teach each topic from the beginning assuming they know nothing. ☐ I know exactly what math the lesson will contain.
Supporting Theory: *Constructivist: students construct their knowledge dynamically through social practices.*	***Supporting Theory:*** *Behaviourist: knowledge is transmitted to students.*

Figure 1. Sample of survey items organized into charts

Figure 1 shows Component 1, also presented in Appendix A, and is the first page of the tool that teachers used to help describe their teaching beliefs and practices. This process involved a participant's self-assessment of where he or she stood on a beliefs and practices scale. Teachers self-assessed their beliefs and practices by rating themselves according to the charts. They were asked to select a position on a continuum between two modes as described in the charts, and then share their stances with the group.

It was intentional, in this study that this exercise was not meant to objectively measure participants' beliefs and practices, but rather to serve as a tool to help teachers self-identify their beliefs and practices. Participants responded to each component (see Appendix A) of mathematics, teaching, learning, and critical pedagogies, and the group's responses were discussed. The teachers were then provided the opportunity to negotiate their beliefs and practices individually and with the group. Following this activity, questions and concerns about the responses were elicited, and the link to power relations was further explored. Specifically, the focus group questions (FG2) were as follows:

1. Were you surprised about any of the results from this group's beliefs?
2. Do you have any concerns, questions or positive comments about this group's beliefs?
3. Do you see the power relationships in our job in a new or different light, or not?

I used open questioning techniques to elicit more detailed information from the participants. This focus group discussion was audio-recorded and transcribed verbatim. One benefit of action research as described by McCutcheon and Jung (1990) is that participants are given the opportunity to examine more formal epistemologies, and have time to become somewhat familiar with what these formal epistemologies of practice have to offer. As previously argued, critical social science practice requires that participants be given choices rather than be guided towards a researcher's predetermined conclusion (Brown, et al., 2007; Kemmis & McTaggart, 2005; Leitch & Day, 2000). An important purpose for this study was to provide participants with information about teaching pedagogies and philosophies that might be new to them. Reflective journaling work for Session II included the participants taking the Component charts of their beliefs with them, and studying their answers carefully to see if, after reflection, they would decide to keep their choices, or change them. The first written journal reflection was based on questions about how they placed themselves on the various continua, and how those responses might be connected to their practice or other aspects of their world of work. The reflective journal questions for J1 were:

1. Would you change your responses to any of the chart items given more time to reflect?
2. What, if anything, do you notice about the three components (teaching, learner, nature of math)?
3. What, if anything, do you notice when you compare your beliefs and your practices?
4. Are you confident about your position in any of the charts?
5. Are you concerned or confused about any of your beliefs?
6. Write two statements for each component explaining why you selected each of the choices you made.
7. Provide any further thoughts about how this exercise may or may not have helped you explain or understand your world of work.

These questions were the participants' first opportunity to show their journaling skills. Participants reflected about their beliefs and practices throughout the sessions of the study. They did so orally, during recorded focus group discussions, and through journal entries. The articulation and negotiation of their beliefs and practices were important steps towards supporting their self-awareness and reflection.

Session III

In Session III, I first facilitated a professional development segment by offering a "constructivist" lesson, and proceeded as if participants were the students and I was the teacher. This session included defining and learning how to support group work in a high school mathematics class. We then built a rubric together that described how we hoped our students would behave as they worked in groups. I then worked with the group on a problem known as the "painted cubes", and I asked the teachers to work in groups of two or three to build the models using plastic linking cubes. The problem involves building a larger 3D cube with smaller cubes, imagining that it is dipped in paint, letting it dry, and then pulling the cube apart to reveal smaller cubes, some of which have paint on one or more surfaces. The teachers worked through this problem as expected. We found many patterns and functions. My goal was to push their pedagogical knowledge as far as reasonable, and I encouraged them to ask questions about why the patterns existed. We ventured into calculus connections, and then stopped the

mathematics lesson to refocus the group after the learning activity. These focus group questions (FG3) were designed to have the participants reflect deeply on the activity they recently engaged in:

1. What did you experience as a learner in this lesson?
2. What are the benefits of teaching/learning this way?
3. What are the barriers?
4. How, if at all, might experiences like this affect your practice?

After the audio-recorded focus group conversations, I described the next assignment. The participants were to *step outside of their comfort zone* and try a lesson that they believed would be empowering for their students. Specifically, I asked them to "plan and deliver one exciting lesson as if there were no barriers". This session was important because "teachers need first-hand experiences working on specific innovative investigations and activities that they are attempting to use in their classrooms" (Wilson & Cooney, 2002, p. 142).

Critical thought cannot occur without critical action, and vice-versa. In her work, which is especially significant because it reveals the effects of critically reflective practice with secondary mathematics teachers, Jaworski (1998) concluded that "[a]ction, a key concept in the research, led thinking initially and became ultimately inseparable from thinking. The teachers' research was described as *evolutionary* to recognise the dialectical relationship between thinking and action as the two developed symbiotically" (p. 26). This finding is valuable evidence that describes how reflective professional learning practices generated a *clarity* between teachers' beliefs and practices. Fay (1987) determined that his $E_1E_2E_3$ model requires "socially transformative action" (p. 205), and that we are driven to action by our enlightenment. In this study, teachers were *acting* as they participated in focus groups, as they worked actively through their journaling processes, and as they designed and implemented a lesson based on choices of epistemological teaching pedagogies.

The participants' journal questions (J2) for this session were:

1. What stands out about your students' responses?
2. What, if anything, surprised you?
3. What went well in your *best* lesson?
4. How could you improve the lesson?
5. How could you improve the beliefs of the students in your class over the long run?

As reflective journaling work (J2), teachers were to write about their students' responses and to plan a short presentation in order to report their experiences back to the group during the next session.

Session IV

During Session IV, each member shared her/his *best lesson* experience. This focus group session (FG4) was also audio-recorded. Participants' names were chosen at random for the presentation order. Each teacher presented his or her lesson, which took almost all of the time for this session. The reflective journaling work was to write about what they learned from their colleagues' presentations. In their journals, they were asked to answer the same power relations questions and *best lesson* questions that were asked in the first focus study session. Specifically, for J3, they were asked:

1. What did you learn from your colleagues' presentations?
2. In my job, who (or what) has power or influence over my work?
3. In my job, over whom (or what) do I have power or influence?
4. What are the biggest barriers that stop me from implementing the best lessons?
5. What are the factors that facilitate my best work as a teacher?

It is important that the teachers reflected directly on what they had recently implemented in the classroom. As M. S. Wilson and Cooney (2002) described: "[i]t is through the act of reflecting on specific events that those centrally held beliefs can be affected in fundamental ways" (p. 142).

At this point, I asked the participants what they would like us to do during our next (and last) session. I offered that we had many choices, that they could either try their lesson again, or try a colleagues' lesson and report on it, or that I could bring them more samples of lessons. They chose to have me present other sample lessons from which they could borrow ideas.

Session V

This final session (FG5) was also audio-recorded. I facilitated a professional development segment using a social justice lesson called *Poverty and World Wealth: Recognizing Inequality* from Gutstein and Peterson (2005, pp. 64-67), and the *Piles of Tiles* activity from Burns, Wickett, and Kharas (2002, pp. 197-221). As a group, we worked through these tasks very quickly to get a sense of the lessons. I then provided a number of other ideas for social justice lessons using the same resource (Gutstein & Peterson, 2005). Ideas included *sweat shop math* - asking students where their clothing comes from, taking chances using the lottery, where that money goes, and cultural issues regarding gambling, Barbie™ and Ken™ doll measurements, Mercator and Peters projections of world maps, and individual and world spending and wealth. My intent was to show the range of possibilities if teachers chose to *get more political* and show students how they could use mathematics as a tool for empowerment. We used the *Piles of Tiles* activity to show a more constructivist activity that could be done before the *Dipping Cubes* activity to introduce students to arithmetic progressions, linear functions and to the idea of generalizing using algebra.

For reflective journaling work (J4), teachers answered the following questions:

1) Have my beliefs changed over the course of this research project? If so, how have they changed?
2) What, if anything, have I gained from using the charts of survey items to pinpoint my beliefs?
3) What, if anything, have I gained from writing reflections?
4) What, if anything, have I gained from participating in reflective discussions with colleagues?
5) How has our group benefited, or not, from the work we have done together.
6) What, if anything, have I gained with respect to how I interpret the social institution in which I work, and how I interpret the traditions and pressures embedded in my work?
7) Do I believe that what I have learned in this study will affect my practice? Why or why not?

Data Sources and Analysis

Data sources included audio recordings of the five 2-hour focus group sessions, 28 written reflective journals, seven lesson plans, and other lesson documents and web resources over the course of the study. I transcribed all audio data within a few days of each session in order to capture the conversations accurately. All transcribed audio-data conversations and journal data were read and re-read with the themes in Table 1 and the research question in mind. Upon immersing myself into the data, sub-themes became evident. The teachers' expressions of Enlightenment, Empowerment and Emancipation in conversation and journal transcripts were coded into the sub-themes using a two-column content analysis method and NVivo*™* software. I summarized coded and categorized focus group and journal data. Once the data were classified according to Fay's three-stage description of the $E_1E_2E_3$ transformative process and according sub-themes that emerged, the data were re-read and re-classified when necessary, according to themes and sub-theme categories that had been clarified and refined through the re-reading process. This produced the coding structure shown in Table 6.

Table 6. Data Themes (Informed by Table 1)

Themes		Sub-Theme Coding
Theme I) Enlightenment (E_1): Becoming Aware	Teachers become aware of their beliefs and practices, and of alternate pedagogies for teaching mathematics.	Teachers examine, articulate, and justify their beliefs and practices. Teachers notice tensions between their beliefs and practices. Teachers notice the impact of self-reflection exercises.
Theme II) Empowerment (E_2): Becoming Critical	Teachers recognize their beliefs within the larger framework of cultural structures. They are driven to action.	Teachers discuss "hard questions", barriers and power relations to implementing change in their practice (e.g., cultural, organizational, and psychological). Teachers begin to see themselves within a larger framework of schooling, and they notice their positioning within that framework, and that things *can* change.

Themes		Sub-Theme Coding
Theme III) Emancipation (E_3): Taking Action	Teachers realize their ability to make changes by taking action in their classroom and reflecting together.	Teachers reformulate their alternate hypotheses or philosophies. Teachers take action by testing their new hypotheses in their classrooms. Teachers reflect on the benefits of collaboration. Teachers know "finally who they are" (Fay, 1987).

The findings are presented in three themes. The first group lists evidence of discovering, examining and awareness of self. This is the E_1 stage according to Fay (1987), and, in this study, it was where participants developed consciousness about their beliefs, practices, and alternate pedagogies for teaching mathematics. The second set of data lists evidence of critical thought, awareness of their work as within society, politics, and a particular teaching culture and other assumptions. According to Fay, this is the empowerment (E_2) stage, and was evident when teachers discussed barriers and power relations, and recognized their beliefs within a larger framework of cultural structures. The third group of data encompasses actions and a sense of who they were as participants. Fay refers to this period as the emancipatory stage, and teachers here recognized their ability to make changes to themselves or to their environment by taking action.

Results

Data Theme 1: Enlightenment – Becoming Aware

The evidence pointed to teachers' emerging awareness that they needed to understand, explore, articulate and justify their own beliefs and practices. This data theme also includes the participants' comments about how they benefited from reflective journaling. The theme describes the stage of becoming aware.

In the reflective journal (J1), Grant explained: "This exercise has helped me to clarify my beliefs" (Grant's Journal - J1), and Cara also noted that she had never thought about certain ideas such as whether or not students were born with an innate ability to do mathematics. She stated: "Many of the questions related to beliefs I felt were *tricky* questions ...there were some questions that I have never really thought about such as '[s]ome people have a knack for learning and others don't'" (Cara's Journal - J1).

Lecia wondered why her practices didn't match her beliefs:

> I think I'm also a little confused with the way that I teach if I believe something contradictory. Why am I teaching the way that I am if I believe that students can learn better by figuring things out on their own as opposed to me just showing them the "right way" to do things? I find that my practices are often very different from my beliefs. Why? (Lecia's Journal - J1)

She was also taken by the first journaling exercise that asked her to justify her beliefs, and enjoyed it even though she found it very difficult:

> I think it helps to explain *what* I'm confused about and maybe why I haven't changed anything yet... I loved doing this exercise. I found it very challenging, but I wanted to really think about all my answers and dig deep to be able to get a lot out of this. Thank you for getting us to do this! (Lecia's Journal - J1)

Teachers were able to make their beliefs explicit, justify them. The most experienced participant explained:

> I've got to admit that this reflection process has been challenging and insightful. It has made me aware that trying something different can be more beneficial for the students. I've never questioned my practices before!! It's the first time someone has asked me why and I had to support my reasoning. (Alex's Journal - J1)

This teacher, who had more than 25 years of experience, reflected that he had never been asked to justify his beliefs. Participants noticed inconsistencies between their beliefs and practices. Julian wondered: "It appears that I am not entirely implementing what I believe, how can I change this?" (Julian's Journal - J1).

In the final journal (J4), when the teachers were asked to look back on which exercises were, in their minds, the most productive. Participants showed some vulnerability and risk-taking in their admissions. Dale, one of the experienced participants, noted:

> The most productive part of this for me is that I was suggesting what I believe. Stating your beliefs, even in a relatively safe anonymous environment makes you take ownership and solidfy [*sic*] why you do what you do. It requires introspection and self-understanding. (Dale's Journal - J4)

Learning to be critically reflective was a difficult task. As one participant wrote:

> To tell you the truth I find writing reflections a little bit annoying because it forces you to look in the mirror and sometimes I don't always like what I see. Although making me feel guilty about myself, it makes me want to change and that might be a good thing. (Raymond's Journal - J4)

Writing a reflective journal was also difficult for most teachers, but they appreciated the benefits. Dale, an established teacher in the group, wrote: "I think reflection time is one of the most under-rated aspects of productive teaching" (Dale's Journal - J4). Cara explained how difficult it was for her to write her ideas, and described how beneficial journaling was when it came to sharing and expressing herself about her beliefs. She wrote: "I do not really enjoy writing; I struggle with getting my thoughts down on paper. The more we have had to write, the easier it has become for me to share my opinions and ideas and reflect on myself" (Cara's Journal - J4).

Julian described the journal writing in this study as being more effective than his last year of schooling (his teacher preparation year), which was current in his mind since he was a teacher candidate on practicum. He explained: "For starters, I really don't like writing journals, reflections, or any kind of paper (that is why I went into math ☺). However, these journals really helped me, more so than my last year of schooling" (Julian's Journal – J4). Interestingly, Raymond, whose belief system was very traditional at the beginning of this study, described how difficult it was for him to be reflective because he found it very hard to admit to his beliefs and practices. In the *E_1: Becoming Aware* stage of Enlightenment, teachers articulated and justified their beliefs and practices, noticed tensions between them, and noticed the impact of the self-reflection exercises.

Data Theme 2: Empowerment – Becoming Critical

When asked questions about power relations and barriers during focus group sessions, the participants discussed and unmasked the pressures and supports that pushed and pulled them in their work. Barriers included cultural, organizational, and psychological concerns such as their own comfort zones. The ability to become critical is to be able to place oneself within these structures, and come to realize that something can be done to improve, in this case, our practice. In this category, teachers also noticed and described how things could change. This is the empowerment stage.

When engaged in journaling exercises and when participating in focus groups, the teachers reflected critically as they developed an awareness of the social constructs that acted as barriers to implementing change in their practices. I was able to question and challenge beliefs about barriers and was careful to support teachers' concerns. A number of barriers were uncovered and discussed during the focus group discussion in Session II. Barriers described by the participants included: (1) not having enough time to cover the curriculum, (2) not having enough preparation time, (3) a concern about the assessment practices of the government bodies that distract from instruction, (4) the problems of low government funding and large classes, and (5) issues of students coming to teachers with low motivation or skills.

Once barriers were made explicit and discussed by the group, teachers were able to put them aside. Although the barriers were ever-present, every teacher in the group chose to plan and implement a lesson that they described as "outside their comfort zone". They engaged in action research and planned and taught a lesson, observed student feedback, and reflected both personally and as a group on their lessons. After the group presentations of their lessons in Session III, Julian described in his journal (J2) his difficulty in relinquishing his presumed role as main source of knowledge in his classroom, and he described the benefits he perceived for the students.

> Giving control over, and stepping away from the *norm* was not easy, but it made the lessons more engaging for both the students and myself. I am going to try and do this more often. The responses, and energy in the room really changes when students think that they are steering the ship (even if you are holding the wheel). (Julian's Journal – J2)

During the journaling exercise (J3) at the end of Session IV, when asked what she learned from her colleagues' presentations of their lessons, Lecia felt a personal tension between her own expectations to deliver perfect lessons, or to take a risk and lose a little bit of control over the outcome. She explained: "Now that I realize that

my best lesson doesn't have to be perfect, I think I am willing to try more things outside of my comfort zone" (Lecia's Journal – J3). Participants in the study were pleased with each other's risk taking and testing of their philosophies in the classroom. As Alex expressed: "I was quite impressed with everyone wandering out of their comfort zones and doing these amazing group investigations" (Alex's Journal – J3).

As a result of beginning to adopt a critically reflective stance, the participants reformulated their hypotheses and showed evidence of being motivated "into socially transformative action" (Fay, 1987, p. 205). The moment when teachers are *nudged* towards taking action is when they can see that they have reformulated their philosophies. This is like an emancipatory catalyst when it occurs. Evidence of participants achieving that understanding can be found in their journals. In his last journal reflection, when asked to look back on the important aspects of his experience, Raymond was self-critical about his practice as he wrote in his journal (J4). He disclosed that he did not allow his students to learn for themselves often enough. He described: "I am more aware of the need to try a constructivist approach to learning in my class more often. Far too often I tell/show my students exactly what I want them to do and they are not given enough opportunities to figure things out for themselves" (Raymond's Journal – J4).

Lecia described her change of consciousness as a critically important event. She stated: "It has awakened me from being stuck in my own way of doing things from year to year" (Lecia's Journal – J4), and later in the same journal entry, she described how she needed a catalyst such as this study to get her started: "I just need to be pointed in a direction and kick-started with some ideas" (Lecia's Journal – J4).

As Lecia explained in her journal (J3):

> [M]y new goal is to implement one best lesson in each unit that I teach...whether it's directly related to the curriculum or not. And if I try to tell myself that I can't afford to lose a class for it, I have to remind myself that I can't afford to not do it either, otherwise what kind of a teacher would I be? (Lecia's Journal – J3)

Cara declared: "I would love to introduce more social justice problems to my students to help empower them in their lives through mathematics. I would also like to adjust my teaching style to follow a more constructivist model where the students can take ownership in their learning" (Cara's Journal – J4).

In the *E_2: Becoming Critical* stage of Empowerment, teachers appeared to benefit from discussing the power relations and barriers inherent in their work. The stage was set for them to consider and discuss barriers, and for them to notice that they have chosen a position within the structure of teaching, which now means that they can chose another. Because the conditions were met for them to notice that things *could* change, they began to shift their beliefs towards what they felt would be more student-empowering practices, and they were able to indicate their plans to take action.

Data Theme 3: Emancipation – Taking Action and Reflecting on their Transformation

The teachers in the study acted on their beliefs, and implemented a lesson that they considered to be a risk-taking experience. They then shared their lesson with the group, and reported on their successes and difficulties. The participants presented rich pedagogical ideas and practices, and important instructional themes became apparent as teachers shared the salient aspects of their lessons. This process of communicative action and collaboration was critical in building trust within the group. This data theme includes examples of participants' reflections about how they were able to take action and make changes in their practice, how the collaborative and public aspect of the group was essential, and how, after this experience, they were able to know themselves better as practitioners.

Action

The action that was taken is mostly evident with respect to the research process itself. Teachers took action by participating *in the public sphere* during focus groups, they implemented lessons that were outside their comfort zone, and they were able to describe their shifts in beliefs and practices.

In her journal reflections (J4), when asked how the study impacted her work, Lecia described how she had progressed in her practice: "I have already implemented more interactive, hands-on, self-discovery lessons in

my classes since I started this project and I know that over time I will add more to build on these lessons" (Lecia's Journal – J4).

Cara was very thoughtful in her description of her personal change. She explained how she learned to appreciate the complexity of teaching, and described her implementation of teaching mathematics through social justice.

> I think that my main belief that has changed over the course of this research project is that there are so many more issues/ problems/ questions that can be involved in and related to the math curriculum. I am able to introduce social justice issues and political issues in a way that can be related to some of the targeted [curriculum learning outcomes]. (Cara's Journal – J4)

Action as a Group

Raymond described what he saw as a change in consciousness of the whole group. He explained how the team of participants in the study benefited from the reformulation of their pedagogical philosophies. Raymond wrote: "Our group has benefited because it has made us more open to discussing alternative methods for teaching mathematics" (Raymond's Journal – J4). The importance of the collaboration between colleagues was hard to deny. Cara's comment is a good example of the experiences of the members of the group.

> The lesson sharing was really beneficial. Simply getting more ideas of lesson plans that vary from the *norm* as a way to intrigue students and motivate their learning. Hearing each other's opinions and struggles is always beneficial. It allows me to realize that I am not alone in my struggle to change and develop new methods of instruction. It is reassuring to know that even teachers with way more teaching experience than I have encounter the same constraints and obstacles. (Cara's Journal – J4)

Grant had also noted: "[m]y colleagues displayed new and intriguing ways to get students interested in math, using games, because all students seem to enjoy games, or at the very least, enjoy fun. The different lessons were interesting and hands on, another aspect that many teenagers enjoy" (Grant's Journal – J3).

Knowing Myself Better

As Kemmis and McTaggart (2005) contended, the participants' communicative action opened the "communicative space…[and built] …solidarity between the people who open their understanding to one another" (p. 576). The participants expressed that the processes in the study had a positive impact on them, and most felt that they knew themselves better as teachers by the end of the study.

> Having done this project, I feel more confident in my choices for my teaching style and more certain of what I want my classroom to look like. (Lecia's Journal – J4)

> It has given me the chance to figure out who I am as a teacher, and what I want to do, or how I want to teach. (Julian's Journal – J4)

During this stage of taking action, teachers' beliefs were more solidified:

> I do feel like I am better equipped to teach in a constructivist way as opposed to a teacher-directed way (like I currently do). I also feel like a constructivist classroom is more attainable than I first thought. After doing this research project, I am more motivated, inspired, and committed to doing more constructivist-type lessons (Lecia's Journal – J4).

After describing how the charts that contained the beliefs items helped him to reflect, Dale described how the activity helped to solidify his beliefs:

> Then, once you know why you believe what you believe or how you feel, you can then try new things to change to further develop yourself. I think it is a good idea to begin with this before you try to begin to reflect. The activities helped to solidify your belief system. (Dale's Journal – J4)

Lecia used the charts in a similar way, and described how it helped her be more true to herself: "I think reflections are always a useful learning tool. I think it makes me more compelled to make change and to be true to myself" She then described how the study made her more sure of herself: "Having done this project, I feel more confident in my choices for my teaching style and more certain of what I want my classroom to look like" (Lecia's Journal – J4).

The appeal of this study was strong for the teacher candidates on practicum. They wrote about how it helped them to gain confidence and strength to be themselves within the complex world of teaching. Grant wrote:

> The writing of reflections has aided me in clarifying my position, but also helped to scrutinize my own teaching practices in light of social issues. This, I believe, will be very constructive, as I am a pre-

> service teacher, I am just developing my methods of instruction, and instead of having to change them, I can generate them using skills I have learned during this research project. This project has helped me to focus on what my teaching principles should be, in line with my beliefs, to modify my instruction techniques to reflect such. (Grant's Journal - J4).

Julian concluded: "It has given me the chance to figure out who I am as a teacher, and what I want to do, or how I want to teach" (Julian's Journal - J4).

In the *E_3: Taking Action* stage of Emancipation, teachers described a shift in philosophical perspective, and they took action by testing those new hypotheses in their classrooms. They felt that they benefited from the action of collaborating together, and described how they knew finally "who they are" (Fay, 1987, p. 205).

Discussion

Towards a New Model for Critical Practice

It was apparent that the methodology and tools implemented in this study had moved the participants to shift their beliefs and practices. The following model highlights the most impactful tools and practices that were used to support the teachers in becoming aware and critical of their beliefs, and that moved them to change their practice.

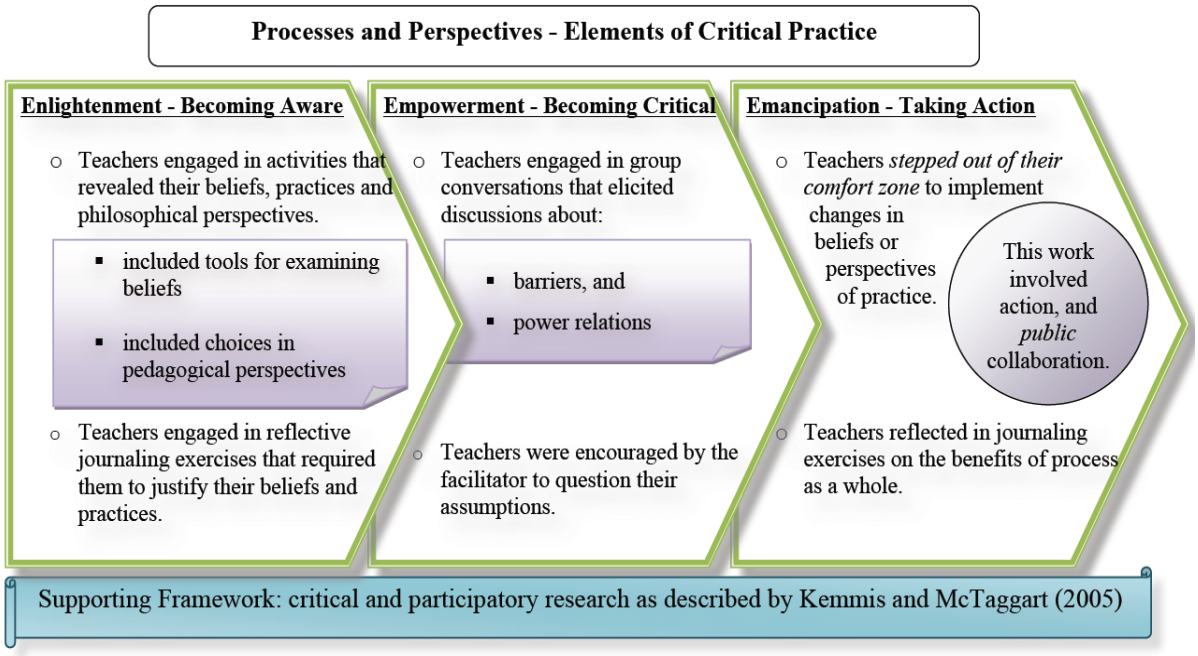

Figure 2. Processes and Perspectives: Elements of critical practice

The three *arrow-shaped* boxes in the Processes and Perspectives Model in Figure 2 describe the three main stages of $E_1E_2E_3$ that teachers experienced in this study. The figure gives a summary of what were the most impactful practices according to the participants. The processes that made a difference for the teachers in this study are listed in three columns, and the supporting perspective and facilitation skills are given in the section at the bottom of the diagram.

This study showed that, by employing a critical and participatory research methodology informed by Kemmis and McTaggart (2005), and by applying Fay's (1987) framework that included three stages of empowerment, teachers were able to become aware of their beliefs, practices and philosophical stances towards teaching mathematics. This is significant because secondary mathematics teachers, in particular, may be impacted by a certain level of expert blindness (Nathan & Koedinger, 2000) that prevents them from considering the assumptions they make about teaching and learning mathematics.

This study showed that it was possible to impact the beliefs and practices of a group of teachers with varying levels of loyalty to more traditional practice. By proceeding through the practices in the Processes and

Perspectives Model in Figure 2, and by ascribing to a critical practitioner's stance, I was able to provide the space for participants to move through Fay's (1987) three stages of $E_1E_2E_3$ to a point where the teachers felt they understood themselves better as educators. These teachers felt aware and confident enough to discuss how they enacted practices that they deemed to be more empowering for their students.

The most impactful processes, as described in the Processes and Perspectives Model are clarified next.

Enlightenment (E_1): Becoming Aware

The first arrowed box on the left side of Figure 2 shows the factors that most influenced the participants' transformation. The secondary mathematics teachers benefited from opportunities to examine, articulate, and negotiate their beliefs about teaching and learning mathematics, and alternate pedagogies. Exploring and having to justify their beliefs about mathematics, teaching, learning, and alternate pedagogies were important first steps. Teachers indicated that the exploratory charts tool used in this study (which can be found in the Appendix) was very effective in helping teachers become aware of their beliefs and practices. The Framework of Perspectives (see Table 5) and the selections of pedagogical perspectives given to the teachers provided choice for the professionals and more freedom and understanding of philosophical perspectives. This was essential in creating a norm of trust and support within the group.

Empowerment (E_2): Becoming Critical

The center arrow box in Figure 2 shows the empowerment (E_2) stage the teachers experienced. The teachers explained that they benefited from the hard questions discussed and the ability to articulate the barriers that stood in their way, and discuss the power relations and institutional narratives that defined and limited them. As Sherer (2008) reminded us: "[w]hile teachers have a lot of control over their classroom practices, they often feel powerless in the larger organizational setting" (p. 17). Teachers' work is complex, and driven by various political and social pressures that may or may not be fully understood by teachers. Eliciting conversations about these pressures and giving them prominence served to recognize the barriers and allowed teachers to then face them. It was also important that I challenged the assumptions that might limit the possibilities for teachers to do their best work. Examples of these could be conversations based on assumptions about teaching, learning, or the subject matter that might justify the status quo, or *how things are* in their work.

Finally, as participants noticed the inconsistencies between their beliefs and practices, and as they made more sense of the field within which they worked, it became important for them to take action. The questions and conversations that took place during the *Becoming Critical* session helped to *nudge* participants towards noticing that they wanted to take action. This advanced them to the next stage.

Emancipation (E_3): Taking Action

The teachers in this study valued the reflective practices that helped them to learn *who they are* as teachers. These processes included implementing lessons, collaborating with colleagues, and applying reflective practices such as journaling. The teachers actively engaged in the research process by reformulating their personal philosophies and learning to describe how their beliefs and practices fit within a framework of pedagogical perspectives. The participants also took action, and implemented lessons that they shared with their colleagues.
As suggested earlier, I have considered practitioner-derived knowledge to be trustworthy and relevant throughout this research project. The teachers in this study opened up to each other during focus group sessions and were willing to share their beliefs, practices and challenges with their colleagues. Their interpretation of what they did in their classrooms is certainly as valid as an outsider's perspective. The teachers in this research project took action by meeting and sharing with their colleagues *in the public sphere*, by implementing new lessons, and by reflecting on how that had impacted their beliefs and practices.

In order to be able to take a risk and implement a lesson, the teachers had to build an understanding of what those *new* practices might be. Teachers identified pedagogical ideas that they considered to be outside of their comfort zone but possibly more valuable for their students. The teachers reported that the time they had to collaborate was an essential element in their learning. They were generous in how they reported their experiences in their classes, and they were honest and shared student feedback and successes with each other. By participating in communicative action together, they formed a bond of trust and professional solidarity.

In order to evaluate the progress of the teachers in the group, I asked them to reflect on the processes they experienced. It was important for teachers to reflect on the benefits of the work they did together in order to sustain or further develop their practice (refer to **Session V** questions on page 206 of this article). Without this opportunity to challenge their beliefs, teachers may have remained unaware of the discrepancy between their beliefs and practices, and of their potential for transformation. In their own words, participants in this study "solidified their belief systems", became "true to themselves", became "more confident of their teaching choices", and figured out "who they are" as teachers. These important emancipatory steps to take action, including putting into practice one's new ideologies, communicating and sharing together in a public space, and maintaining reflective practices were critical steps in completing Fay's $E_1E_2E_3$ stages of critical practice.

Conclusion

This paper described a theoretical and practical framework for supporting secondary mathematics teacher practice. The study was viewed from the perspective of teachers who, through their action and reflective engagement, advanced through the stages of empowerment as described by Fay (1987). It was designed to address Kemmis and McTaggart's (2005) seven key research features with the goal of bridging the dichotomy between research and practice as Kemmis and McTaggart had described. By applying a perspective that was informed and analyzed using Fay's stages of Enlightenment, Empowerment, and Emancipation, critical and participatory research tools were implemented to provide the space for teachers to become aware, become critical, and take action. This paper presented research and practice tools such as charts for examining beliefs, questions for negotiating and justifying those beliefs, group and individual questions that elicited conversations about barriers and power relations in teachers' work, and an overall process for critical and participatory research that provides a space where the participants can critically reflect on their beliefs and practices, step out of their comfort zone, apply a lesson, and discuss it collaboratively with their colleagues.

The Processes and Perspectives Model brings together theory and practice from established literature. Within the framework of Kemmis' and McTaggart's (1988, 2005) participatory action research, I have included the theoretical constructs provided by Fay's (1987) $E_1E_2E_3$ model of enlightenment-empowerment-emancipation, Smyth's (1989a, 1989b) *hard* questions, and Lerman's (2003) pedagogical modes (used to design the FoP Framework of Perspectives – Table 5). The Processes and Perspectives Model (Figure 2) presents strategies for researchers and practitioners to apply practical tools within robust theoretical constructs that serve the needs of those participating in the process. Research studies such as this one, that regard practitioner-derived knowledge as valuable and are framed through a teacher empowerment and critically reflective research perspective, hold promise in contrast with the more common reform and prescriptive research models.

Implications

Though this study introduced teachers to various philosophies and pedagogies for teaching and learning mathematics, more work could be done with these teachers to deepen their understanding and to explore more thoroughly the implications of applying their perspectives in their classrooms. Longer-term professional development and more opportunities to implement lessons involving a *professional learning community* would likely be beneficial for the teachers who participated in this study. Perhaps, should funding be available to release teachers during the day, a *lesson study* approach (e.g., Stigler & Hiebert, 1999) could be applied so that teachers could observe each other as they teach. This could provide a more generative environment for teachers to reflect personally and collaboratively about what it means to teach and learn mathematics from various philosophical perspectives.

Repeating this study would fundamentally depend on the perspective and practices of the facilitator. The researcher/facilitator would have to understand the intent of Fay's description of the empowerment process, and would have to hold a perspective on praxis that is consistent with that of a critical practitioner and, in particular, hold an understanding of research as anticipated by Kemmis and McTaggart (2005). Simply applying the tools or procedures described in this work without an understanding of how to *be* a critical researcher would not be sufficient. Through praxis, the "emancipatory interplay between action and reflection" (McCutcheon & Jung, 1990, p. 147), the critical theorist facilitator can apply the Processes and Perspectives Model presented in this study to reexamine the "taken-for-granted constraints … of schooling" (p. 147) and provide the space for teachers to better understand and impact their practice.

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Appendix A: Charts with Organized Items

Component 1: My View of Teaching	
Competence Mode:	Performance Mode:
My Goal: ☐ My goal in class is to help students construct mathematical knowledge and to empower them as learners and thinkers.	***My Goal:*** ☐ My goal in class is to help students practice and master important mathematical procedures and skills.
My Beliefs: ☐ Teachers should encourage students to find their own solutions to problems even if they are inefficient. ☐ Teachers should allow students who are having difficulty solving a problem to continue to try to find a solution. ☐ Teachers should facilitate students' inventions of ways to solve problems.	***My Beliefs:*** ☐ Instructors should focus on facts instead of theories. ☐ The teacher should demonstrate how to solve simple word problems before students are allowed to solve problems. ☐ An effective teacher demonstrates the right way to do a problem. ☐ Teachers should tell students who are having difficulty solving a problem how to solve it.
My Practices: ☐ I find out which parts students already know and don't teach those parts. ☐ I teach students differently according to their individual needs.	***My Practices:*** ☐ I teach the whole class at once. ☐ I tell students which questions to tackle. ☐ I go through one method for doing each question. ☐ I teach each topic from the beginning assuming they know nothing. ☐ I know exactly what math the lesson will contain.
Supporting Theory: *Constructivist: students construct their knowledge dynamically through social practices.*	***Supporting Theory:*** *Behaviourist: knowledge is transmitted to students.*

Component 2: My View of The Learner	
Competence Mode:	Performance Mode:
My Goals: ☐ In my teaching, building self-confidence in learners is a priority.	***My Goal:*** ☐ I expect students to master a lot of information related to the subject.
My Beliefs: ☐ Students learn through discussing their ideas. ☐ Even students who have not learned basic skills and procedures can have effective methods for solving problems. ☐ Students learn best by figuring out for themselves the ways to find answers to problems. ☐ In my class(es), students pose mathematical questions and develop problems based on their own interests. ☐ Most of all, learning depends on what a student already knows.	***My Beliefs:*** ☐ Some people just have a knack for learning and others don't. ☐ Frequent practice on skills and procedures are essential in order for students to learn them. ☐ Most students cannot figure out for themselves how to solve problems and must be explicitly taught. ☐ Students learn best from teachers' demonstrations and explanations. ☐ Students should master procedures and skills before they are expected to understand how they work.
My Practices: ☐ In my class(es), students compare different methods for doing questions. ☐ I encourage students to challenge each others' thinking. ☐ In my class(es), students work collaboratively in pairs or small groups. ☐ In my class(es), students invent their own methods. ☐ In my class(es), students choose which questions they tackle.	***My Practices:*** ☐ In my class(es), students' written answers to paper-and-pencil mathematical problems are used to indicate their level of understanding. ☐ In my class(es), students demonstrate their learning through doing exercises.
Supporting Theory: *Constructivist: students construct their knowledge dynamically through social practices.*	***Supporting Theory:*** *Behaviourist: knowledge is transmitted to students.*

Component 3: My View of The Nature of Mathematics	
Competence Mode:	**Performance Mode:**
My Goal: ☐ It is not my instructional goal to use mathematics for the purpose of addressing social justice issues.	***My Goal:*** ☐ It is not my instructional goal to use mathematics for the purpose of addressing social justice issues.
My Beliefs: ☐ Mathematics exists because humans construct it. ☐ What is true is a matter of opinion. ☐ Truth about mathematics changes as the mathematics community interprets it in a new light.	***My Beliefs:*** ☐ What is true today will be true tomorrow. ☐ Mathematics exists and we discover it. ☐ If two people are arguing about something, at least one of them must be wrong. ☐ Mathematics is value-free.
My Practices: ☐ I challenge familiar ways of understanding the subject matter.	
Supporting Theory: *Constructivist: students construct their knowledge dynamically through social practices.*	***Supporting Theory:*** *Behaviourist: knowledge is transmitted to students.*

Two Other *different* Competence Modes!	
Competence Mode III: *Criticalmathematics* (Emancipatory)	**Competence Mode II: *Ethnomathematics* (Populist)**
My Goal: Political Empowerment ☐ My goal in class is to help students learn to use mathematics as a tool for political empowerment.	***My Goal: Cultural Empowerment*** ☐ My goal in class is to help students learn to use mathematics as a tool for cultural empowerment.
My Beliefs: ☐ In my class(es) I want to make apparent what people take for granted about society.	***My Beliefs:*** ☐ In my class(es) I want to make apparent what people take for granted about society.
My Practices: ☐ In my classes, students pose political questions that require a mathematical approach.	***My Practices:*** ☐ In my classes, students pose culturally relevant social justice questions that require a mathematical approach.
☐ I use the subject matter as a way to teach about higher ideals. ☐ I link instructional goals to changes that should be made in society. ☐ My teaching is governed more by social issues than by the course objectives.	☐ I use the subject matter as a way to teach about higher ideals. ☐ I link instructional goals to changes that should be made in society. ☐ My teaching is governed more by social issues than by the course objectives.
Supporting Theory: *Critical Theory: knowledge is a set of critical competencies, and students take a critical stance towards knowledge.*	***Supporting Theory:*** *Sociology: knowledge is a set of critical competencies that allow the challenging of socially constructed boundaries.*